Universal Gas Constant

Manuel Antoinette, Patrick Ponce

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Abstract

We calculated a value for the universal gas constant R by measuring the mass of air of a known volume. We obtained the mass by weighing an air-filled and an empty glass piston and calculating it's difference. We calculated the volume by weighing the piston while filled with water, the density of which we know from literature. Our calculations resulted in a value of $R = 8.0\,\mathrm{J\,K^{-1}\,mol^{-1}}$, which is 3.6 % off the literature value. A further calculation taking humidity into account did not yield a better result, most likely due to too inaccurate measurements.

1 Introduction

We can describe an ideal gas using the following pieces of information: mass m, molecular weight M_r, temperature T, volume V, and pressure p. It has been shown empirically that, these five variables are related via the universal gas constant R:

$$\frac{pVM_r}{mT} \approx R = (8.31434 \pm 0.00035)\frac{\mathrm{J}}{\mathrm{mol} \cdot \mathrm{K}}$$

In this lab we aimed to calculate the value of R by measuring the mass of air of a known volume.

2 Experiment

Our main tool was a glass piston as depicted in Figure 1. We began by measuring the mass m_{KL} of the glass piston filled with air. We then sucked out the air with a rotary vane pump and created a vacuum inside of the glass piston, then sealing it by melting the glass together at the end. We then measured the mass m_{KV} of the empty glass piston. Next, we filled the piston with water, let it rest alongside a cup of water, the temperature of which we could measure, until they both reached similar temperatures t_w. We let it rest for about 30 minutes. Then we once again measured the mass m_{KW_a} of the water-filled glass piston. Finally we took measurements of the air temperature t, humidity φ and pressure p using a mercury barometer.

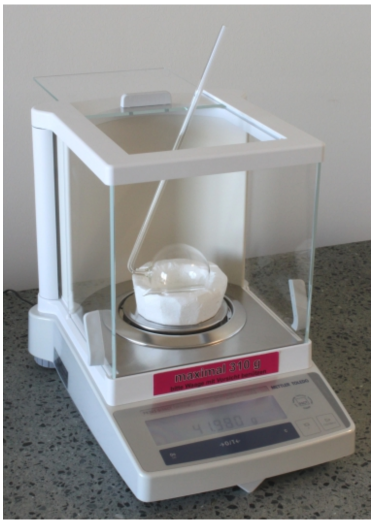

Figure 1: Glass piston placed on a scale to measure it's mass.

3 Results

The Mass of the air inside the glass piston is given by the mass difference of the air-filled piston $m_{KL} = (49.396 \pm 0.002)$ g and the empty piston $m_{KV} = (49.182 \pm 0.002)$ g, which gives us $m = (0.214 \pm 0.004)$ g. Analogously, with the difference of the mass of the water-filled piston $m_{KWa} = (238.550 \pm 0.002)$ g and the empty pistons mass, we can calculate the mass of the water inside the piston: $m_{Wa} = m_{KWa} - m_{KV} = (189.369 \pm 0.004)$ g.

Using a literature value for the water density $\rho_{Wa} = 0.997\,79\,\mathrm{g\,mL^{-1}}$ for our measured water temperature $t_w = (21.9 \pm 0.1)\,°\mathrm{C}$ we could calculate the volume V inside the piston using $V = \frac{m_{Wa}}{\rho_{Wa}}$. From the mercury barometer we read a value of $b' = (698.83 \pm 0.10)$ mmHg, which results in a (wet) air pressure of $p = (93\,169 \pm 13)$ Pa. With this, we are ready to make our first approximation for the universal gas constant. By using the literature value $M_L = 28.96\,\mathrm{g\,mol^{-1}}$ for the molecular weight of air and the formula

$$R \approx \frac{M_L p V}{m T},$$

where $T = t + 273.15 = (196.95 \pm 0.50)$ K is the room temperature. This gives us a value of $R = (8.05 \pm 0.16)\,\mathrm{J\,K^{-1}\,mol^{-1}}$ for the universal gas constant.

A more accurate result can be calculated by taking the humidity in the air into consideration and using

$$R = \frac{M_L p_L V}{m_L T}, \tag{1}$$

where p_L and m_L denote the partial pressure of the dry air and mass of the dry air respectively. Using the Magnus formula to calculate the saturation vapor pressure p_{WS} and our measurement of the relative humidity $\varphi = (42.0 \pm 0.5)\,\%$ we can calculate p_L using

$$p_L = p - p_W = p - \varphi p_{WS} = p - \varphi A e^{st/(T_n+t)},$$

where p_W is the humidity's partial partial pressure and $A = 611.2\,\mathrm{Pa}$, $s = 17.62$, $T_n = 243.12\,°\mathrm{C}$ are given. This gives us $p_L = 91833^{+53}_{-145}$ Pa. The mass of the dry air is given by

$$m_L = m - m_W = m - \frac{M_W p_W V}{RT}, \tag{2}$$

where m_W is the mass of the water in the air and $M_W = 0.018\,015\,\mathrm{kg\,mol^{-1}}$ is the molecular weight of water taken from literature. As we can see, equation (2) for m_L contains the universal gas constant R. So, to calculate R we have to insert (2) into (1) to get a formula for R with humidity taken into consideration:

$$R = \frac{V(M_L p_L + M_W p_W)}{mT}$$

With this, we get a value for the universal gas constant of $R = (8.00 \pm 0.17)\,\mathrm{J\,K^{-1}\,mol^{-1}}$.

4 Discussion

We obtained fairly accurate results, being that the measured gas constant was only about 3.6% off of the literature value for the gas constant. There were many variables, which could have contributed to the error, for example the room temperature measurement T, the water temperature measurement T_{Wa}, and the mass of the water in the glass piston m_{KWa}. We measured a difference of $1.8K$ from one side of the room to the other, possibly from a draft in the room, and therefore cannot be sure how exact the measurements we used were. Secondly, it took a long time to fill up the water piston with water, during which we held it the whole time and therefore might have influenced the temperature before we measured it, possibly affecting the measurement of the density of the water. Although we were diligent in the filling of the water piston, we might not have gotten every single air bubble out, which also could have affected the "more accurate result" when taking humidity into consideration.

5 Conclusion

We measured the mass of the air inside of a glass piston using a scale and calculated its volume by weighing it while filled with water. With our measurements we calculated the universal gas constant R to be around $8.0\,\mathrm{J\,K^{-1}\,mol^{-1}}$, which is about 3.6 % off the correct value from literature of $0.314\,\mathrm{J\,K^{-1}\,mol^{-1}}$. When taking humidity into account and performing a more complex calculation, we did not obtain a better value. Reasons for this might include inaccurate measurements of the humidity, air- and water temperature.