4 – Transversal Oscillation of a String

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Abstract

In the first part we measured the resonance curve of a 42 cm long iron string. We did this by driving the string with an electromagnet at a fixed frequency (127.7 Hz) and then measuring the amplitude of the oscillating string for various tensions. In the second part we measured the resonant tensile force (at the same driving frequency as before) for strings of various lengths. From our measurements we then experimentally determined the resonant frequency to be (127.29 ± 0.05) Hz, which is within 0.4% of the driving frequency.

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1 Introduction

We define a string as a flexible cord, whose cross section is small compared to its length l. Therefore, if we bend the string, there is almost no moment that counteracts the bending. If a string is fixed at both ends, deformation will only cause work to be done against the tensile force $\vec{Z}$ acting in the longitudinal direction of the string.

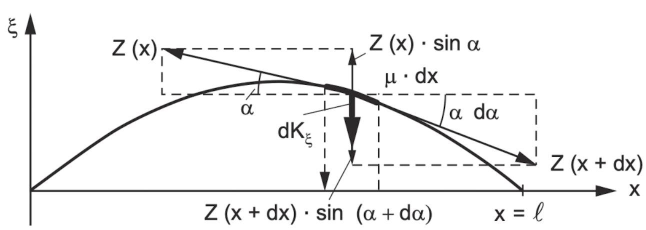

Figure 1: Diagram of forces at the position x of a deflected string. The restoring force dK_ξ can be calculated from the tensile force $Z(x)$ along the string.

In a small string element of length dx there exists a restoring force $dK_\xi = Z(x+dx)\sin(\alpha+d\alpha) - Z(x)\sin\alpha$, as can be seen in figure 1. For small deformations, we can use the approximations

$Z(x+dx) \approx Z(x)$ and $\sin\alpha \approx \tan\alpha \approx d\xi/dx$. With this we get the relation

$$dK_\xi = Z\left[\left(\frac{d\xi}{dx}\right)_{x+dx} - \left(\frac{d\xi}{dx}\right)_x\right] = Z\frac{\partial^2\xi}{dx^2}dx.$$

Ignoring the impact of other forces like friction, the restoring force is equal to the force applied to move the piece of string $dm\frac{\partial^2\xi}{\partial t^2}$. We can write $dm = \mu dx$, where μ is the mass per unit length and we get the equation of motion

$$\frac{\partial^2\xi}{\partial x^2} = \frac{\mu}{Z}\frac{\partial^2\xi}{\partial t^2}.$$

We are interested in solutions which fulfill the boundary conditions

$$x = 0: \quad \xi(0,t) = 0$$
$$x = l: \quad \xi(l,t) = 0$$

for all times t and will use the ansatz

$$\xi(x,t) = A\sin(kx)\cos(\omega t),$$

which describes a standing wave with amplitude A, wave vector k and angular frequency ω. When we substitute this into the equation of motion, we get a relationship between the parameters k and ω

$$k^2 = \frac{\mu}{Z}\omega^2.$$

The boundary conditions can be satisfied if we require that $k_n l = n\pi$. With this, we can compute the eigenfrequencies of the string

$$\nu_n = \frac{\omega_n}{2\pi} = \frac{k_n}{2\pi}\sqrt{\frac{Z}{\mu}} = \frac{n}{2l}\sqrt{\frac{Z}{\mu}},\ n \in \mathbb{N}. \tag{1}$$

In this lab, we will be observing the fundamental frequency

$$\nu_1 = \frac{1}{2l}\sqrt{\frac{Z}{\mu}}, \tag{2}$$

which represents a free undamped harmonic oscillator. We will force an iron wire into transverse oscillation by periodically changing the magnetic field $H = H_0\sin(2\pi\nu t)$. First, we will be measuring the amplitude while keeping the string at a fix length and only changing the tensile force. Then, we will vary the length of the string and measure the tensile force needed for the string to be at resonance.

2 Experiment

Resonance Curve We measured the resonance curve of a 42 cm long string. For this we fixed both ends of the string and applied the tensile force on it by hanging weights onto a lever mechanism, which pulls on the string. Note that the lever arm had a 2:1 ratio, which means that the tensile force was twice as large as the weight force of the masses. An electromagnet then forced the iron string to oscillate at exactly 127.7 Hz. Using a magnifying lens and a cross hair, which had an accuracy of 0.01 mm, we then measured the amplitude of the oscillation for various tensile forces.

Tensile force for varying lengths With the same setup as in the previous experiment, we now varied the length of the string from 36 cm up to 54 cm in 2 cm increments. For each length we carefully set the tensile force such that the amplitude of the oscillation is maximal.

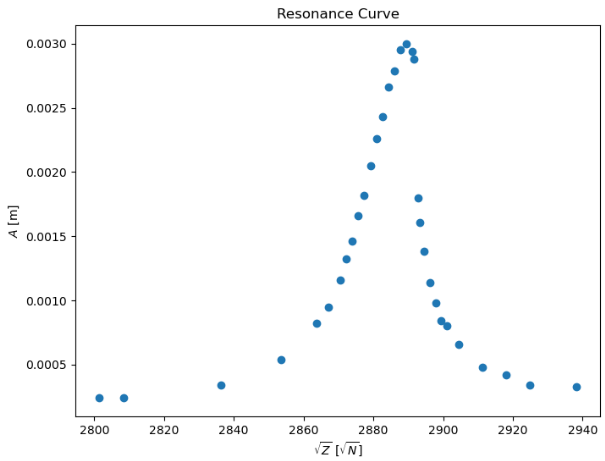

Figure 2: Resonant curve of the string with length $l = 42\,\text{cm}$. We plotted the amplitude A as a function of the square root of the tensile force Z.

3 Results

Resonance Curve Figure 2 plots the measured amplitude against the square root of the tensile force of the string. The tensile force Z is given by

$$Z = 2Mg,$$

where M is the mass hanging from the lever and $g \approx 9.81$ is the acceleration due to earths gravity. Note the factor of 2 due to the lever arm.

From theory we'd expect a symmetric Gaussian-like curve. Our measurements are clearly not symmetric and slightly slanted to the right. Unfortunately, we could not identify any problems in our setup or data analysis. Also note the gap in data points on the upper right side of the curve. Even with our finest mass increment $(0.2\,\text{g})$ we could not get into that range. If the resonance curve is truly slanted, this would make sense, as the curve would be very steep in that region.

Tensile force for varying lengths Figure 3 plots the resonant tensile force Z against the square of the length l^2. From 1 get

$$Z = 4\mu\nu_1^2 \cdot l^2.$$

Therefore the linear relation ship is in accordance with the theory. From the slope s of the fit we can also calculate an experimental value for the resonant frequency ν as follows:

$$\nu = \sqrt{\frac{s}{4\mu}} \qquad \Longrightarrow \qquad \nu_{\text{exp}} = (127.29 \pm 0.05)\,\text{Hz}$$

Comparing this to the driving frequency of $127.7\,\text{Hz}$, we can see that we're less than 0.4% off.

4 Discussion

As stated in the results section, the measured resonance curve is slightly slanted, even though in theory we would expect a Gaussian curve. This could be due to too inaccurate measurements. Had we used $0.05\,\text{g}$ increments, perhaps we would have measured a better curve. In order to obtain better results on how the resonance curve looks in general, we could have measured it for different lengths of string as well and then compared all the shapes. We were able to accurately

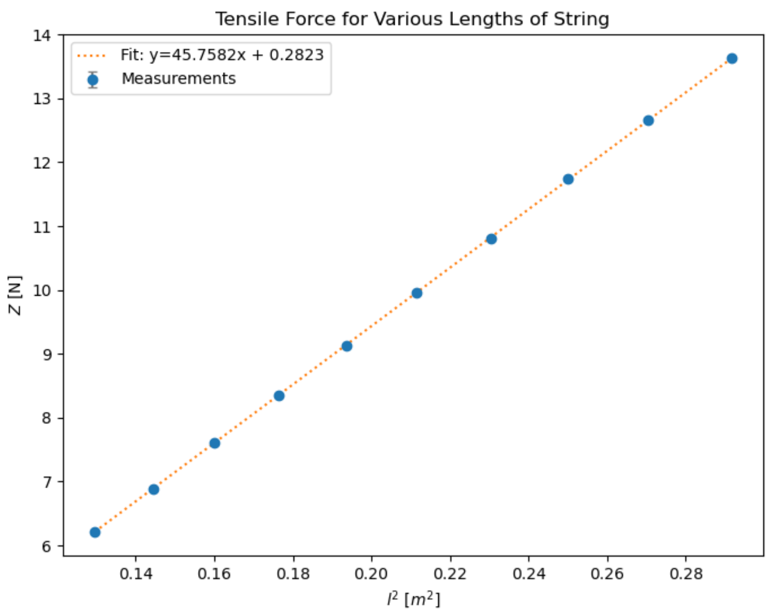

Figure 3: Depicted is the tensile force Z at resonance frequency for various lengths of string l ranging between 36 and 54 cm. We plotted the tensile force as a function of the length squared.

measure the tensile force for various string lengths at resonance, since our measured value for the resonance frequency was less than 0.4% off of the driving frequency 127.7 Hz. We could have however obtained more accurate measurement, had we used smaller masses to measure the tensile force. There were some cases in which we were not able to discern a difference in amplitude when adding or removing 0.2 g.

5 Conclusion

In conclusion, we were able to measure the resonance curve for the fundamental oscillation of a 42 cm long iron string. We also verified equation 2, since the measured resonance frequency $\nu_{\text{exp}} = (127.29 \pm 0.05)\,\text{Hz}$ was very close to the driving frequency of the magnetic field 127.7 Hz.