Spectroscopy

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Abstract

In the experiments conducted, we determined the wavelengths of the lines in the Balmer series of the Hydrogen spectrum, which can be found in table 2. We calculated the grating constant $g = (3.25 \pm 0.09)\,\mu\text{m}$ of the grating used in the spectrometer. We then verified Bohr's theory, by calculating the Rydberg frequency $Ry_{\text{exp}} = 3.22 \times 10^{15}\,\text{s}^{-1}$ and the ionization energy $E_{\text{ion}}^{\text{exp}} = 13.3\,\text{eV}$.

1 Introduction

Bohr's atomic model states that an atom is comprised of a Z-fold positively charged core, where Z is the reference number, with radius $r \approx 10^{-14}$m, that is surrounded by electrons, which orbit the core, like planets orbit the sun. Using this concept, for the electron to stay in orbit, the centripetal and coulomb forces must cancel each other out, which leads to the equation

$$\frac{m_e v^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2},$$

where m_e is the mass of an electron, v is its velocity, r is the distance from the core, e is the elementary charge, and ε_0 is the vacuum permeability. From this we can calculate the kinetic and potential energy, whih gives us the total energy

$$E_{tot} = E_{kin} + E_{pot} = \frac{mv^2}{2} - \frac{kZe^2}{r},$$

with $k = \frac{1}{4\pi\varepsilon_0}$.

According to quantum physics, only discrete levels of energy are possible. So if an electron changes energy levels, a proton must be emitted or absorbes. The frequency of this converted light is $\nu = \frac{E_1 - E_2}{h}$. It follows that no radiation takes place on stationary orbits. This leads us to the quantization of angular momentum $L_n = n\hbar$. Bohr's quantum condition is therefore

$$m_e v r_n = n\hbar,$$

which leads us to $r_n = \frac{\varepsilon_0 h^2 n^2}{\pi m_e e^2}$. The total energy is then

$$E_n = -\frac{k^2 Z^2 e^4}{2\hbar^2 n^2}.$$

The energy emitted when an electron goes from a higher to lower energy level is then for $n < m$

$$\Delta E = E_2 - E_1 = \frac{m_e e^4}{8\varepsilon_0^2 h^2}\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$$

and therefore

$$\nu = \frac{c}{\lambda} = Ry\left(\frac{1}{m^2} - \frac{1}{n^2}\right), \tag{1}$$

where $Ry = \frac{m_e e^4}{8\varepsilon_0^2 h^3}$. According to Huygen's principle, the phase difference of two neighboring waves, which go through a grate with groove spacing g is given by $\Delta = g \sin\varphi$, where φ is the angle of the point from the incident light. The condition for a brightness maximum for the wavelength λ is then $p\lambda = g\sin\varphi$, where p is a integer called the order number of the diffraction maximum. The brightness maxima in a spectrometer can be calculated with

$$p\lambda_c = g(\sin\varphi_{in} - \sin\varphi_{out}).$$

When the grating is at the rotational angle α_d, this turns into

$$p\lambda_c = 2g\sin\alpha_d\cos\alpha_m,$$

where $\alpha_m = 15°$ and $p \in \mathbb{Z}/\{0\}$.

2 Experiment and Results

2.1 Calibrating the Spectrometer

Our first task was to calibrate the spectrometer. We did this by illuminating the spectrometer with light from an He lamp and measuring at what knob position r_c each bright line in the He spectrum is placed in the central pixel of the CCD. The knob position for each wavelength observed can be found in table 1.

We linearly fit our data to get an equation of the form $\lambda_c = ar_c + b$, where λ_c is the wavelength at the center of the screen, r_c is the knob position and a and b are determined by a linear regression, which can be seen in figure 1. This gives us the following equation, which relates the central wavelength λ_c with the knob position r_c:

$$\lambda_c = 0.441r_c + 442 \pm 10$$

Color	r_c [-]	λ_c [nm]	Color	r_c [-]	λ_c [nm]
Dark Red	625	728.1	Blue 2	9	443.7
Red 1	553	706.5	Blue 3	-6	438.8
Red 2	483	667.8	Violet 1	-53	416.9
Yellow	406	587.6	Violet 2	-75	414.4
Light-Green	179	504.7	Violet 3	-77	412.1
Green 1	167	501.6	Violet 4	-122	396.4
Cyan	141	492.2	Dark-Violet	-134	388.9
Blue 1	20	447.1			

Table 1: In this table we have the knob position for each line observed in the He spectrum. The values of the wavelength were given in the protocol sheet attached. The errors for the wavelengths is half of the last digit (e.g. (728.10 ± 0.05) nm) and for the knob positions is estimated to be around 10 (e.g. (625 ± 10)).

2.2 Calibrating the Pixel Scale of the CCD Camera

The second task was to calibrate the pixel scale of the CCD camera by fitting the pixel position x of a bright line in the He spectrum to the knob position r with a linear dependence $(x - x_c) = a'(r - r_c)$, where x_c and r_c denote the central pixel position and the corresponding central knob position respectively.

We decided to do this calibration with the yellow line with a wavelength of 587.6 nm. We took images of the line at 5 different positions x, including the central position x_c. For each of the 5 images, after averaging the pixel values along the vertical axis and the three color channels, we fitted the (now one dimensional) pixel values on the horizontal axis with a gaussian profile. The maximum of the gaussian profile gives us the pixel position x for each image. Our measurements are shown in figure 2. Fitting this graph with a linear function, gives us a value of $a' = -13.3 \pm 10$. Therefore the final equation to determine the wavelength λ of a line at position x on the screen, where the wavelength λ_c of the central pixel position x_c are known is:

$$\lambda = \lambda_c - \frac{0.441}{13.3}(x - x_c) \tag{2}$$

2.3 Determining the Grating Constant

Using the equation $p\Delta\lambda \approx -\Delta x(g/f)\cos\alpha_m$, we can calculate g, using the values $p = 1$, $f = 38.1$ cm and $\alpha_m = 15°$. For Δx and $\Delta\lambda$ we took the measurements from table 1 and calculated their difference in wavelength and pixel position relative to the central line *Blue 1* at a wavelength of 447.1 nm. The Δx is calculated using the calibration of the pixel scale established previously. These gave us an average value for the grating constant with the standard deviation for the error:

$$g = (3.25 \pm 0.09)\,\mu\text{m}$$

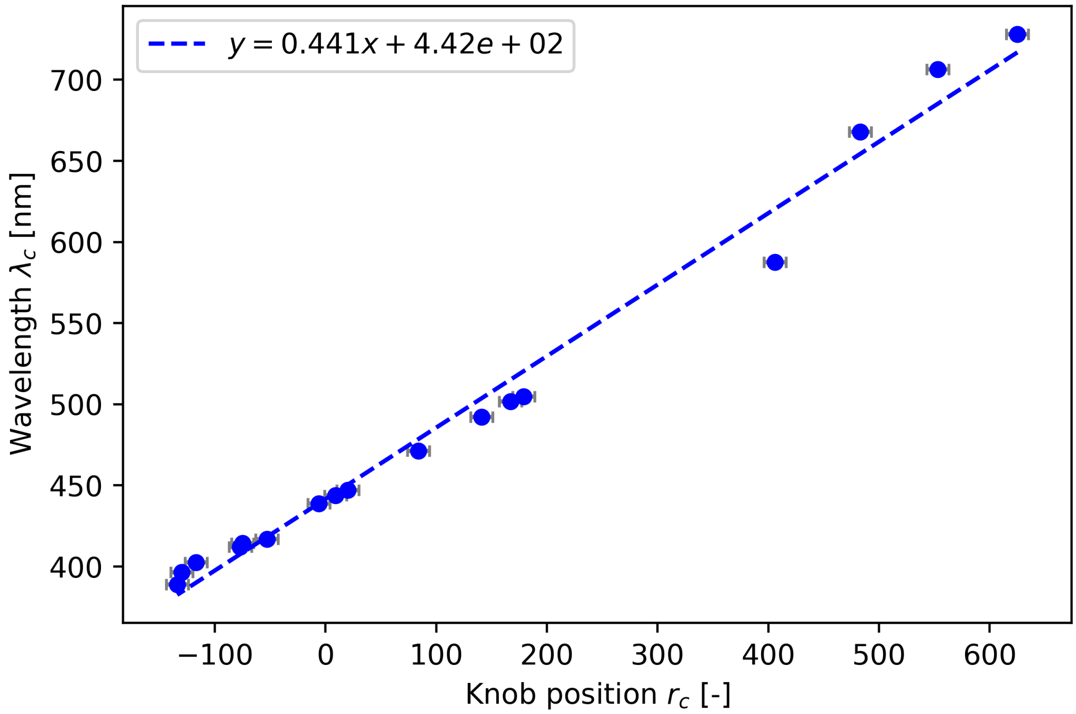

Figure 1: The graph shows the data in table 1 fitted with a linear function $\lambda_c = ar_c + b$.

2.4 Wavelength of the Balmer Series of the Hydrogen Spectrum

In this experiment we observed the Hydrogen spectrum. For each observed bright line, we recorded the knob position, as shown in table 2. We then calculated the corresponding wavelengths with the established calibration of the spectrometer and equation (2). These wavelengths can also be seen in table 2 along with some literature values. We measured the FWHM by taking pictures of each of the wavelengths observed and fitting them to a Gaussian profile. In table 2 you can see the measured values of the FWHM compared to the theoretical values, which were calculated using the formula

$$\Delta\lambda_{\text{Theor.}} = g\lambda/l,$$

where we assume the size of the whole grating $l = 50\,\text{mm}$ is illuminated by the incoming light.

2.5 Verification of Bohr's theory

To verify Bohr's theory, we observe equation (1), where $m = 2$ and $n \in 3, 4, 5, ...$ for the Balmer series. The values for ν, $\frac{1}{m^2} - \frac{1}{n^2}$, and Ry_{exp} can be found in table 3. The average Rydberg frequency is $Ry_{\text{exp}} = 3.22 \times 10^{15}\,\text{s}^{-1}$ and the literature value of the Rydberg

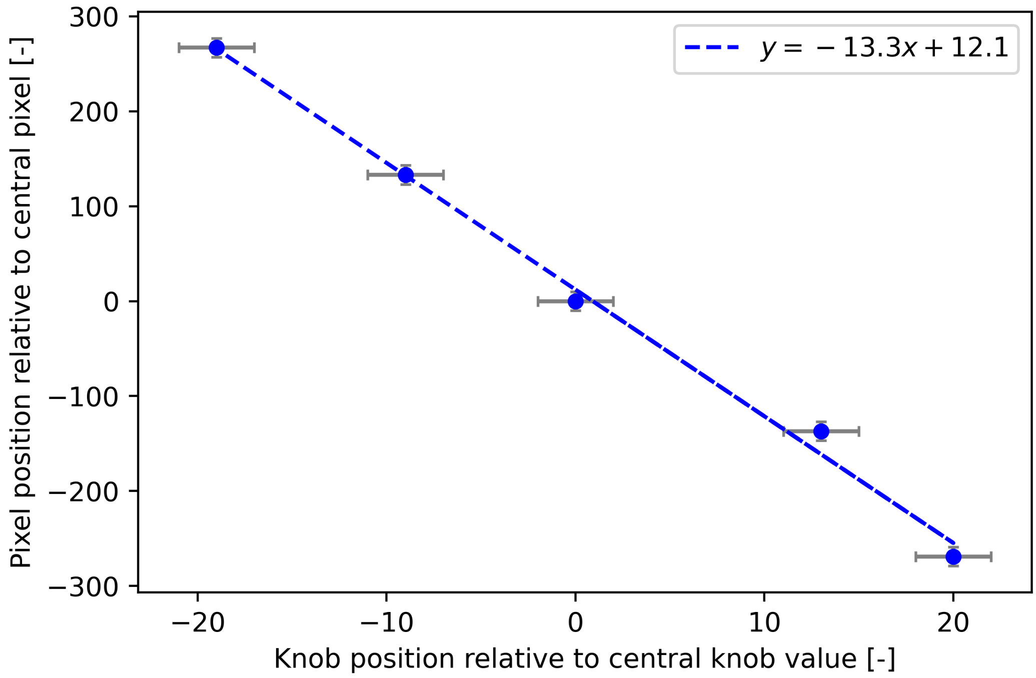

Figure 2: The graph shows our measurements on how the horizontal pixel position is related to the knob position, fitted with a linear regression.

frequency is $Ry_{\text{Lit.}} = 3.29 \times 10^{15}\,\text{s}^{-1}$.
To calculate the limit of the Balmer series, we let $n \to \infty$ in equation (1), which leads to

$$\nu = \frac{c}{\lambda_{\text{lim}}} = \frac{Ry_{\text{exp}}}{4} \implies \lambda_{\text{lim}} = \frac{4c}{Ry_{\text{exp}}}.$$

With our average value for the Rydberg frequency, we get $\lambda_{\text{lim}} = 372.08\,\text{nm}$. We were not able see the limit of the Balmer series, because it lies in the ultraviolet range and the camera sensitivity is low at those wavelengths.
To calculate the ionization energy E_{ion}, we assume that we are ionizing from the ground state $m = 1$ and let $n \to \infty$. Therefore using equation (1), and since $E = h\nu$, we get the relation

$$E_{\text{ion}} = \text{h}\nu = \text{h}Ry_{\text{exp}},$$

where h is Planck's constant. Using the average Rydberg frequency we get $E_{\text{ion}}^{\text{exp}} = 2.14 \times 10^{-18}\,\text{J} = 13.3\,\text{eV}$ and the literature value is $E_{\text{ion}}^{\text{Lit.}} = 13.6\,\text{eV}$.

Color	r_c [-]	λ [nm]	$\lambda_{\text{Lit.}}$ [nm]	FWHM [px]	$\Delta\lambda$ [nm]	$\Delta\lambda_{\text{Theor.}}$ [nm]
Red	598	705.0	656.2	48.2	39.7	45.7
Blue	105	487.8	481.1	31.5	25.9	31.7
Purple	-29	428.8	430.0	27.4	22.6	27.8

Table 2: The table shows the color of the three brightest lines in the hydrogen spectrum, the knob position r_c such that the they are centered on the screen, the corresponding calculated wavelengths λ, which have an error of $\Delta\lambda \pm 20\,\text{nm}$, and some corresponding literature values taken from this link. Included in the table are also the measured FWHM of each wavelength as well as the theoretical values.

Color	n	$\nu[10^{14}\text{nm}]$	$\frac{1}{m^2} - \frac{1}{n^2}$	$Ry_{\text{exp}}[10^{15}\text{s}^{-1}]$
Red	3	4.25	0.14	3.06
Blue	4	6.15	0.19	3.28
Purple	5	7.00	0.21	3.33

Table 3: The table shows the values for the frequency ν of the observed wavelengths in the Hydrogen spectrum, their corresponding energy levels n and Rydberg frequencies Ry_{exp}.

3 Data Analysis

We calculated the error of linear regressions using the root mean standard error formula:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(\tilde{y}_i - y_i)^2}{n}},$$

where $\tilde{y}_i$ is the predicted value for knob position r_c and y_i the is the measured value at the same knob position. This resulted in the errors $\Delta a = 10$ for the linear regression of the knob position in regards to the wavelength and $\Delta a' = 10$ for the knob position relative to the pixel position.
To calculate the error for the grate constant, we used the Gauss propagation method. The equation for the error of the distance between the colors relative to *blue 1* $(\Delta x)_{\text{err}}$ is

$$(\Delta x)_{\text{err}} = \sqrt{(\Delta r_c)^2(\Delta a)^2 + a'^2(\Delta r_c)_{\text{err}}},$$

where $(\Delta r_c)_{\text{err}} = 20$ and $\Delta a' = 10$.
The equation for the error of the grating constant is

$$\Delta g = \sqrt{\left(\frac{pf}{\Delta x \cos\alpha_m}\right)^2 (\Delta\lambda)^2_{\text{err}} + \left(\frac{p\Delta\lambda f}{(\Delta x)^2 \cos\alpha_m}\right)^2 (\Delta x)^2_{\text{err}} + \left(\frac{p\Delta\lambda}{\Delta x \cos\alpha_m}\right)^2 (\Delta f)^2},$$

where $(\Delta\lambda)_{\text{err}} = 0.1$ and $\Delta f = 0.05$. We calculated the error for each wavelength, and took the average error resulting in $\Delta g = 0.09\,\mu\text{m}$.
We calculated the error for the wavelengths in the Hydrogen spectrum by adding the errors of λ_c and the second linear regression performed, giving us an error of $\Delta\lambda = \pm 20\,\text{nm}$.

4 Discussion

We gathered the data using the spectrometer and the CCD, which minimizes the error greatly. Our calculated grate constant $g = (3.25 \pm 0.09)\,\mu\text{m}$ lies withing the realm of uncertainty to the expected grate constant of $3.33\,\mu\text{m}$.
The measured wavelengths for the blue and purple light observed in the Hydrogen spectrum lie within error to the literature values. The ration of the difference between the measured wavelength and literature value for the red light is about 2.5, which is relatively small. This also makes sense, because the measurement for the red light in the Helium spectrum was further off form the first linear regression performed than blue and purple were. The values for the FWHM of each of these colors is lower than the theoretical values, which makes sense, because we are limited by the resolution of the camera and the theoretical value assumes the entire grating is illuminated by the light. We can see that when comparing the Rydberg frequency and ionization energy from our results with the literature values. The difference between Ry_{exp} and $Ry_{\text{Lit.}}$ is $\Delta Ry = 0.07 \times 10^{15}\,\text{s}^{-1}$, which is about 2% of the literature value. The difference between $E_{\text{ion}}^{\text{exp}}$ and $E_{\text{ion}}^{\text{Lit.}}$ is $\Delta E_{\text{ion}} = 0.3$, or also about 2% of the literature value.

5 Conclusion

In conclusion, we were able to accurately measure the observable wavelengths from the Hydrogen and Helium spectrum. The FWHM of each color observed in the Hydrogen spectrum correspond well to the theoretical values. The grating constant, Rydberg frequency, and ionization energy calculated from our results lie close to the literature values. The results could be improved with a more accurate grating knob and the use of a higher resolution CCD.