Specific Charge of the Electron

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Abstract

We calculated the charge to mass ratio of the electron e/m, by both calculating and directly measuring the magnetic field required for an electron beam inside of a Braun tube (cathode-ray tube) to perform exactly one revolution of a spiral. When calculating the magnetic field from the current it is induced by, we obtained a value of $(-1.760\,000 \pm 0.000\,001) \times 10^{11}\,\mathrm{C\,kg^{-1}}$ for e/m, which is within 0.1 % of the literature value $1.759 \times 10^{11}\,\mathrm{C\,kg^{-1}}$. When we tried to measure the magnetic field with a second, smaller, measuring coil and a galvanometer our calculations resulted in a worse value of $(-2.174\,000\,0 \pm 0.000\,000\,1) \times 10^{11}\,\mathrm{C\,kg^{-1}}$, which is 23 % off literature. We believe this deviation occurred, because the galvanometer could not be very accurately calibrated.

1 Introduction

It is difficult to measure the charge e and the mass m of an electron directly, but their ratio e/m can easily be determined via the Lorenz force in a magnetic field $\vec{B}$ and electric field $\vec{E}$:

$$\vec{F} = -e(\vec{E} + \vec{v} \times \vec{B}).$$

We can control or measure the variables in the Lorenz force by performing an experiment with a Cathode-ray tube (CRT), sometimes also called a Braun tube. The electrons in the CRT are deflected and therefore their velocity vector is not parallel to the longitudinal axis of the brown tube. By placing the CRT inside of a solenoid, which produces a magnetic field parallel to the longitudinal axis of the CRT, we create a Lorentz force with magnitude $|F| = |ev_nB|$, where v_n is the component of the electron's velocity normal to the magnetic field. The Lorentz force causes the electrons to move in a spiral. By using the geometry of the spiral, and adjusting the deflection so that the electron performs one spiral before reaching the florescent screen at the end of the CRT, we can calculate the centripetal force $Z = m\omega^2/l$ on the electron, where l is the length of the CRT and ω is the rotation frequency, which is equal to the magnetic component Lorenz force $F = el\omega B$. From this we can determine e/m as follows:

$$\frac{e}{m} = \frac{8\pi^2}{\mu_0} \frac{V}{l^2 H^2},$$

where V is the acceleration voltage and H the magnetic field the tube is placed inside.

2 Experiment

Our setup was comprised of a Braun tube placed inside of a coil producing a magnetic field parallel to the longitudinal axis of the tube. This causes the electrons to fly in a spiral, as their initial velocity contains a component perpendicular to the magnetic field, which is induced by two sets of deflection plates inside the Braun tube. The tube's acceleration voltage V was held constant throughout all experiments. The current I flowing though the surrounding coil could be very finely adjusted up to 0.001A, which gave us precise control over the magnetic field inside the Braun tube.

In our first experiment we adjusted the the current I such that the electron ray spiral inside the braun tube hit the fluorescent screen after exactly one spiral. To get an accurate measure of this, we performed 10 measurements with positive current, and 10 measurements with negative current, effectively changing the direction of the magnetic field. Each 10 measurements were split into two, where each lab partner performed a batch of 5 measurements at a time.

The first experiment resulted in a difference of $(0.0071 \pm 0.0030)\,\mathrm{A}$ between the positive current $\langle I^+ \rangle$ and the negative current $\langle I^- \rangle$. The signal to noise ratio is 2.4, which shows that the value of both currents lie close to each other. The reason for this discrepancy is that the electrons are also being affected by earth's magnetic field.

The measurements of the total magnetic field inside the coil is subject of our second experiment. For this we remove the coil from the Braun tube and place it around a smaller measuring coil. This measuring coil, a galvanometer and a resistor R are then connected in series, in a loop, to form a circuit. If now a switch-on current is applied to the surrounding coil, a magnetic field is created inside the coils, which induces a current I_i into the measuring coil. By measuring the ballistic deflection α of the galvanometer, we can calculate the magnetic field inside the coil. We took 5 measurements of α for the switch-on, and 5 for the switch-off current of $\langle I^+ \rangle$, $\langle I^- \rangle$ and $\langle I \rangle = \frac{1}{2}(\langle I^+ \rangle + \langle I^- \rangle)$, so in total 30 measurements.

Finally we had to determine the ballistic constant b of the Galvanometer. For this we again take the two coils, still placed inside of each other, and connect the galvanometer along with a resistance R, a capacitor C, switch, a voltage source and a voltmeter to the measuring coil to form a circuit. The circuit is build such that when the switch is off, the capacitor gets charged by the voltage source. Once the switch is turned on all the charge Q, which we can calculate, flows through the measuring coil, the galvanometer and the resistor R. This allows us to calculate the ballistic constant b for the circuit used in the second experiment.

3 Results

For the first experiment we set the acceleration voltage to $V = (875 \pm 2)\,\mathrm{V}$. The Voltage slightly drifted throughout the experiment, but never dipped below $873\,\mathrm{V}$ or exceeded $877\,\mathrm{V}$. The average coil current values for when the electron beam was focused were $\langle I^+ \rangle = (0.880 \pm 0.003)\,\mathrm{A}$ and $\langle I^- \rangle = (0.887 \pm 0.002)\,\mathrm{A}$ for the positive and negative current respectively. The errors are standard deviations of all the samples we took. Next we can try to calculate the magnetic field H inside of the coil by using the formula for the magnetic field of a cylindrical coil given by

$$H(I) = nI \frac{1}{\sqrt{1 + \left(\frac{r}{L/2}\right)^2}},$$

where $L = (0.260 \pm 0.005)\,\mathrm{m}$ is the length, $n = N/L = \frac{1629}{(0.260 \pm 0.005)\,\mathrm{m}} = (6265.380 \pm 0.005)\,\mathrm{m}^{-1}$ is the number of windings per meter, $r = (3.820 \pm 0.005) \times 10^{-2}\,\mathrm{m}$ is the average radius of the coil and the average $\langle I \rangle = \frac{1}{2}(\langle I^+ \rangle + \langle I^- \rangle) = (0.884 \pm 0.003)\,\mathrm{A}$ is used for I. This yields us a calculated value for the magnetic field inside the coil of

$$H_{\mathrm{calc}} = (5314 \pm 18)\,\mathrm{A\,m}^{-1},$$

where the error was propagated by using the maximum/minimum values for the input parameters. If we instead use $\langle I \rangle = \frac{1}{2}(\langle I^+ \rangle - \langle I^- \rangle) = (0.004 \pm 0.002)\,\mathrm{A}$ we can calculate the vertical component of the earth field to be

$$H_n^E = (21 \pm 18)\,\mathrm{A\,m}^{-1}.$$

In our second experiment we tried to measure this value directly. For the three currents $\langle I^+ \rangle$, $\langle I^- \rangle$ and $\langle I \rangle$ we measured α-values of $\langle \alpha^+ \rangle = (41 \pm 2)\,\mathrm{mm\,m}^{-1}$, $\langle \alpha^- \rangle = (42 \pm 2)\,\mathrm{mm\,m}^{-1}$ and $\langle \alpha \rangle = (41 \pm 2)\,\mathrm{mm\,m}^{-1}$ respectively. We can determine the magnetic field H using the formula

$$H = \frac{1}{w \mu_0 S} k \alpha, \tag{$\star$}$$

where $w = 395$ is the number of windings of the measuring coil, $\mu_0 = 1.257 \times 10^{-6}\,\mathrm{kgms}^{-2}\mathrm{A}^{-2}$ is the vacuum permeability, $S = (2.390 \pm 0.005) \times 10^{-3}\,\mathrm{m}^2$ is the area enclosed by one winding and $k = (R + R_g)\,b$ with $R_g = (810 \pm 10)\,\Omega$ and $R = (10.0 \pm 0.5)\,\mathrm{k}\Omega$ and b the galvanometer's ballistic constant, is a constant value that we determined during the calibration of the galvanometer.

During the calibration of the galvanometer we used a resistor with resistance $R = 10\,\mathrm{k}\Omega$, a capacitor with capacitance $C = (63.60 \pm 0.05)\,\mathrm{nF}$ and a voltage source of $V = (10.270 \pm$

0.005) V. Further, the combined resistance of the galvanometer and the measuring coil was $R_g = (810 \pm 10)\,\Omega$. With this setup we measured an average displacement of $\langle \alpha_e \rangle = (46 \pm 2)\,\mathrm{mm/m}$. The charge Q_g that has flowed through the galvanometer is proportional to the deflection α_e via the ballistic constant b. With the total charge on the capacitor $Q = CV = Q_g \frac{R+R_g}{R}$ we get a formula for the constant value k:

$$k = b\,(R + R_g) = \frac{CVR}{\alpha_e}.$$

This gives us a value of $k = (1.40 \pm 0.06) \times 10^{-4}\,\mathrm{C}\,\Omega$, which we can plug into ($\star$) to calculate the measured magnetic field:

$$H_{\text{meas}} = (4800 \pm 300)\,\mathrm{A\,m^{-1}}.$$

Now using the formula

$$\frac{e}{m} = \frac{8\pi^2}{\mu_0^2} \frac{V}{l^2 H^2},$$

which can be derived by setting the Lorenz force and the centrifugal force of the spiral equal, we can finally calculate e/m, both using H_{meas} and H_{calc}:

$$(e/m)_{\text{calc}} = (-1.760\,000 \pm 0.000\,001) \times 10^{11}\,\mathrm{C\,kg^{-1}}$$

$$(e/m)_{\text{meas}} = (-2.174\,000\,0 \pm 0.000\,000\,1) \times 10^{11}\,\mathrm{C\,kg^{-1}}$$

4 Data Analysis

To calculate the total error in our measurements, we used the Gauss propagation method: $\Delta f = \sqrt{\sum_i (\frac{\partial f}{\partial x_i})^2 \Delta x_i^2}$. The error in the calculation of the magnetic field is given by:

$$\Delta H_{calc} = \sqrt{(\frac{\partial H}{\partial I})^2 \Delta I^2 + (\frac{\partial H}{\partial n})^2 \Delta n^2 + (\frac{\partial H}{\partial L})^2 \Delta L^2 + (\frac{\partial H}{\partial r})^2 \Delta r^2}$$

$$= \sqrt{(\frac{n}{{\sqrt{1+(\frac{2r}{L})^2}}^2})^2 \Delta I^2 + (\frac{I}{{\sqrt{1+(\frac{2r}{L})^2}}^2})^2 \Delta n^2 + (\frac{4Inr^2}{(1+(\frac{2r}{L})^2)^{3/2} L^3})^2 \Delta L^2 + (\frac{4Inr}{(1+(\frac{2r}{L})^2)^{3/2} L^2})^2 \Delta r^2}.$$

For the calculation of the magnetic field inside of the solenoid, where $\Delta I = 0.003\mathrm{A}, \Delta n = 0.5$ windings per meter, $\Delta L = 0.005\,\mathrm{m}$, and $\Delta r = 0.000\,05\,\mathrm{m}$, we get an error of $\Delta H_{\text{calc}} = 18\,\mathrm{A\,m^{-1}}$. For the calculation of the Earth's magnetic field, with errors $\Delta I = 0.002\mathrm{A}, \Delta n = 0.5$ windings per meter, $\Delta L = 0.005\,\mathrm{m}$, and $\Delta r = 0.000\,05\,\mathrm{m}$, we get an error $\Delta H_n^E = 18\,\mathrm{A\,m^{-1}}$

The error in the calculation of the galvanometer constant k is:

$$\Delta k = \sqrt{(\frac{\partial k}{\partial C})^2 \Delta C^2 + (\frac{\partial k}{\partial V})^2 \Delta V^2 + (\frac{\partial k}{\partial R})^2 \Delta R^2 + (\frac{\partial k}{\partial \alpha_e})^2 \Delta \alpha_e^2}$$

$$= \sqrt{(\frac{VR}{\alpha_e})^2\Delta C^2 + (\frac{CR}{\alpha_e})^2\Delta V^2 + (\frac{CV}{\alpha_e})^2\Delta R^2 + (\frac{CVR}{\alpha_e^2})^2\Delta\alpha_e^2}.$$

For our calculation of k we had the errors $\Delta C = 5 \times 10^{-11}\,\mathrm{C}, \Delta V = 0.005\mathrm{V}, \Delta R = 5 \times 10^{-6}\,\Omega, \Delta\alpha_e = 2\,\mathrm{mm\,m^{-1}}$, we get the total error $\Delta k = 6 \times 10^{-6}\,\mathrm{V\,s\,mm\,m^{-1}}$.
The total error for the calculation of the measured magnetic field is given by:

$$\Delta H_{meas} = \sqrt{(\frac{\partial H}{\partial k})^2\Delta k^2 + (\frac{\partial H}{\partial \alpha})^2\Delta\alpha^2 + (\frac{\partial H}{\partial w})^2\Delta w^2 + (\frac{\partial H}{\partial S})^2\Delta S^2}$$

$$= \sqrt{(\frac{\alpha}{n\mu_0 S})^2\Delta k^2 + (\frac{k}{n\mu_0 S})^2\Delta\alpha^2 + (\frac{k\alpha}{n^2\mu_0 S})^2\Delta n^2 + (\frac{k\alpha}{n\mu_0 S^2})^2\Delta S^2}.$$

The errors associated to the measured magnetic field are: $\Delta k = 6\times 10^{-6}\,\mathrm{V\,s\,mm\,m^{-1}}, \Delta\alpha = 2\,\mathrm{mm/m}, \Delta w = 0.5$ windings, and $\Delta S = 5\times 10^{-6}\,\mathrm{m^2}$, which gives us a total error $\Delta H_{\mathrm{meas}} = 300\,\mathrm{A\,m^{-1}}$.
We define $s = e/m$ as the specific charge of the electron. The total error is given by:

$$\Delta s = \sqrt{(\frac{\partial s}{\partial V})^2\Delta V^2 + (\frac{\partial s}{\partial l})^2\Delta l^2 + (\frac{\partial s}{\partial H})^2\Delta H^2}$$

$$= \sqrt{(\frac{8\pi^2}{\mu_0 l^2 H^2})^2\Delta V^2 + (\frac{16\pi^2 V}{\mu_0 l^3 H^2})^2\Delta l^2 + (\frac{16\pi^2 V}{\mu_0 l^2 H^3})^2\Delta H^2}.$$

The associated errors are $\Delta V = 0.005\,\mathrm{V}, \Delta l = 0.005\,\mathrm{m}, \Delta H_{\mathrm{meas}} = 300\,\mathrm{A\,m^{-1}}$, and $\Delta H_{\mathrm{calc}} = 18\,\mathrm{A\,m^{-1}}$ which lead to total errors $\Delta(e/m)_{\mathrm{calc}} = 1000\,\mathrm{C\,kg^{-1}}$ and $\Delta(e/m)_{\mathrm{meas}} = 10\,000\,\mathrm{C\,kg^{-1}}$ respectively.

5 Discussion

When looking at our value for $(e/m)_{\mathrm{calc}}$ calculated with H_{calc}, we see that we're within 0.1 % of the literature value of $1.759 \times 10^{11}\,\mathrm{C\,kg^{-1}}$[1]. As for $(e/m)_{\mathrm{meas}}$ we see a deviation of about 23.6 %, which means that our measurement of the magnetic field H_{meas} was not very accurate. The difference between H_{meas} and H_{calc} is about 10 %, which most likely is because of the measurements we took with the galvanometer. We set the error for each measurement to $\Delta\alpha = 2\,\mathrm{mm\,m^{-1}}$, because the start value of the galvanometer was not

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[1] 2018 CODATA Value: electron charge to mass quotient. The NIST Reference on Constants, Units, and Uncertainty (link, accessed November 11th 2021).

perfectly centered at $0\,\mathrm{mm\,m^{-1}}$, but displaced by about $1\,\mathrm{mm\,m^{-1}}$. On top of that we added the average error calculated from the standard deviation of our values, which gives us the resulting error. The difference between H_{meas} and H_{calc} is $(600 \pm 300)\,\mathrm{A\,s^{-1}}$ We can see that the ratio between the error and measurement error is very low, and therefore the results are not very far apart.

Finally, we calculated the earth's magnetic field's vertical component to be $H_n^E = (21.34 \pm 18.00)\,\mathrm{A\,m^{-1}}$, which is only about 18 % off from a literature value of $25.97\,\mathrm{A\,m^{-1}}$ taken from the NCEI magnetic field calculator[2] by inputting the coordinates of our lab and the appropriate time. Given that the earths magnetic field varies greatly with location and time, our measurement is certainly realistic.

6 Conclusion

By calculating the magnetic field induced by the coil, we were able to determine the charge to mass ratio e/m of the electron to be $(-1.760\,000 \pm 0.000\,001) \times 10^{11}\,\mathrm{C\,kg^{-1}}$, which is within 0.1 % of the literature value. Though when we tried to directly measure the magnetic field we deviated by about 23 %, ending up with a value of $(-2.174\,000\,0 \pm 0.000\,000\,1) \times 10^{11}\,\mathrm{C\,kg^{-1}}$. We believe this deviation to appear, because the Galvanometer was could not be very accurately calibrated. Finally we measured the earth's magnetic field's vertical component to be $(21.34 \pm 18.00)\,\mathrm{A\,m^{-1}}$, which is within 18 % of a value obtained from literature.

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[2] National Centers for Environmental Information Magnetic Field Calculator (link, accessed November$^{11\text{th}}$, 2021)