14 – Refractive Index of Air

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Abstract

Using an Interferometer of Jamin, we measure the refractive index of air. The interferometer works by splitting a monochromatic light ray into two and letting one of the rays pass through air of different pressure and afterwards leading them back together to form an interference pattern. By observing how this pattern changes for various pressures we calculate the refractive index of air at ambient pressure ((952.80 ± 0.05) Pa) to be $n_T = 1.000253 \pm 0.000007$ at (23.5 ± 0.5) °C and $n_0 = 1.000275 \pm 0.000008$ at 0 °C.

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1 Introduction

The speed of light in a medium u is defined as

$$u = \frac{1}{\sqrt{\varepsilon\varepsilon_0\mu\mu_0}},$$

where ε_0 is the vacuum permittivity constant, μ_0 the vacuum permeability constant, and ε and μ are the permittivity and permeability material properties. In Vacuum $\varepsilon = \mu = 1$. Additionally, for materials in the visible light range one can set $\mu = 1$, therefore we can define the refractive index n as follows:

$$\frac{c}{u} = \sqrt{\varepsilon} = n.$$

It is generally hard to measure the speed of light in materials, therefore one can measure the shift in wavelengths instead, using $u = \nu\lambda$. In gases the permittivity is generally given by

$$\varepsilon = 1 + \frac{1}{\varepsilon_0}\kappa\alpha,$$

with κ being the number of particles per cubic meter and α describing the polarizability of the molecules. Further using the ideal gas law we can describe the refractive index of a gas via a pressure coefficient Λ_T:

$$n = 1 + \Lambda_T p, \tag{1}$$

where

$$\Lambda_T = \frac{N\alpha}{2\varepsilon_0 RT} = \frac{\text{const.}}{T}. \tag{2}$$

We will determine the refractive index of air using an Interferometer of Jamin and measuring the shift in wavelength for various pressures of air. For this we will count the number of interference fringes Z that appear for a given change in pressure p. If the pipe is completely evacuated, the number of wavelengths λ_0 in the pipe of length l is $q = l/\lambda_0$. If the pressure increases the wavelength λ changes and a certain number of fringes Z will appear. The following relation holds:

$$q + Z = l/\lambda.$$

Therefore, after some derivations, we can determine the refractive index n and the pressure coefficient Λ_T as follows:

$$n(p,T) = 1 + \frac{Z}{l}\lambda_0 \tag{3}$$

$$\Lambda_T = \frac{Z\lambda_0}{pl}. \tag{4}$$

2 Experiments

We used an Interfermometer on Jamin as sketched in figure 1. The length of the tube was $l = (165.8 \pm 0.1)$ mm and we used a monochromatic light with wavelength $\lambda_0 = (589.30 \pm 0.05)$ nm. At first the tube was set to ambient pressure. Then, we slowly decreased the pressure inside the tube, which changes the refractive index of the air inside. Therefore the interference pattern seen through the aperture started to move slowly. For every 5 fringes that entered the view, we recorded the pressure inside the tube. We did this by taking a video of the pressure sensor and then giving a visual cue once every five fringes. Then later we looked at the video and wrote down the pressures at every visual cue. Once the tube has reached near vacuum, we closed it off and repeated the same measurement, but backwards. For this we used a capillary, which allowed us to slowly let air back into the tube in a controlled manner.

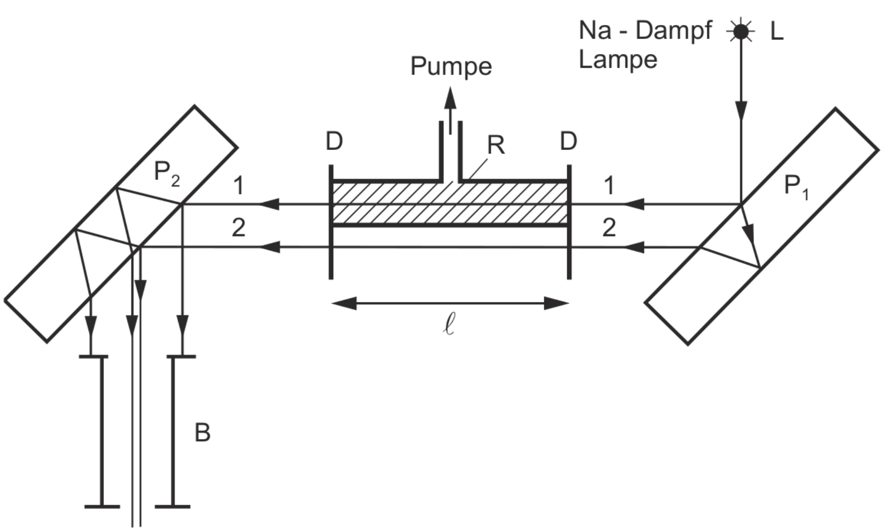

Figure 1: Interferometer of Jamin

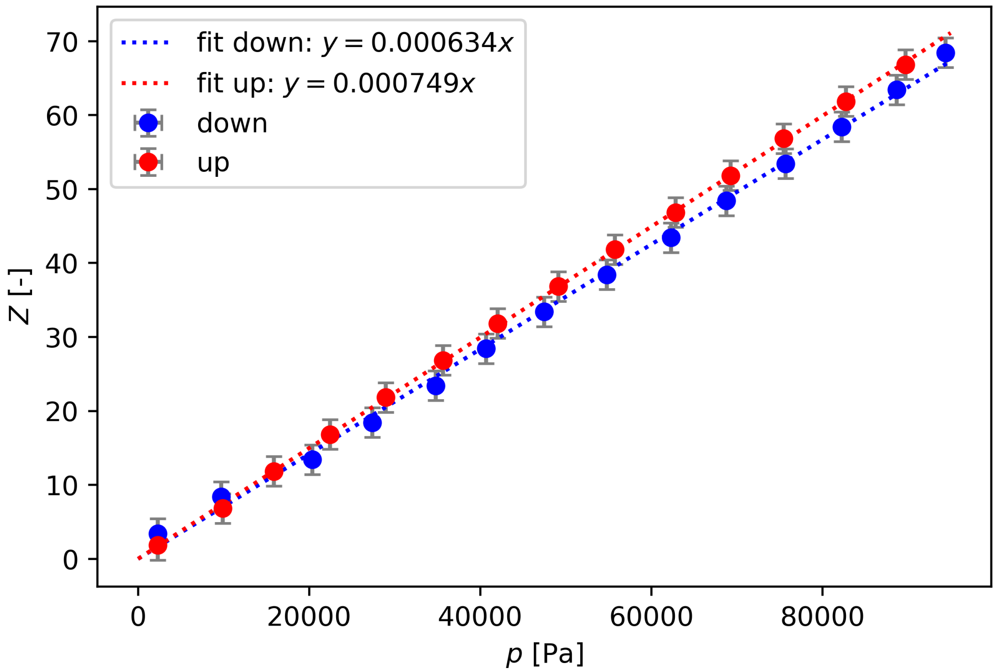

Figure 2: Measurements taken from the interferometer of Jamin. Note that the measurements are shifter vertically, such that the fit passes through the origin. The error vertical error is estimated to be $\Delta Z = 2$. Errors in pressure are smaller than the datapoints.

3 Results and Discussion

Our Measurements are plotted in figure 2 as follows: Each measurement run, from ambient to vacuum (down) and from vacuum to ambient (up) is fitted linearly. As we only know the difference in Z between each measurement (five fringes), we have to vertically align the measurements in such a way, that a theoretical measurement at 0 Pa[1], would land at $Z = 0$. Therefore we shifted the measurements such that the linear fit had displacement of zero.

Theoretically both measurements should overlap perfectly, but as we can see, the *down* measurements have a slightly smaller slope. Reason for this could be that we miscounted the fringes from the third lowest to the second lowest data point, as the last two points are slightly shifted parallel to the rest of the measurements. Errors like these we accounted for in the error bars of $\Delta Z = 2$. The error for the pressure, estimated to be around $\Delta p = 10\,\mathrm{Pa}$, is too small to be shown in the graph.

In the following we are going to perform all calculations with the data from the *up*- and *down*-measurements separately. We believe though, that the results from the *up*-measurement are more accurate, due to the errors discussed above.

Using both fits we can calculate Z_a for ambient pressure p_a and temperature T, which we measured to be

$$p_a = (95\,280 \pm 5)\,\mathrm{Pa}$$
$$T = (23.5 \pm 0.5)\,^\circ\mathrm{C} = (296.65 \pm 0.50)\,\mathrm{K}.$$

Then, with (4) and (3) we can calculate the refractive index n_a and the pressure coefficient Λ_T at ambient pressure and temperature. Finally, using (2) we can determine the constant term and

[1] With our setup it was not possible to get pressures low enough, that we could assume a vacuum. The lowest pressures we could achieve were 23 Pa.

then calculate the pressure coefficient Λ_0 at $0\,^\circ\mathrm{C} = 273.15\,\mathrm{K}$ to determine the refractive index n_0 at zero degrees with (1). All calculations, with $\lambda_0 = (589.30 \pm 0.05)\,\mathrm{nm}$ and $l = (165.8 \pm 0.1)\,\mathrm{mm}$, are shown in table 1. We can compare the refractive indices to literature values

	down	up
Z_a	67.5 ± 2	71.3 ± 2
Λ_T	$(2.52 \pm 0.07) \times 10^{-9}\,\mathrm{Pa}^{-1}$	$(2.66 \pm 0.07) \times 10^{-9}\,\mathrm{Pa}^{-1}$
n_T	1.000240 ± 0.000007	1.000253 ± 0.000007
Λ_0	$(2.74 \pm 0.08) \times 10^{-9}\,\mathrm{Pa}^{-1}$	$(2.89 \pm 0.08) \times 10^{-9}\,\mathrm{Pa}^{-1}$
n_0	1.000261 ± 0.000008	1.000275 ± 0.000008

Table 1: Values calculated from the measurements in figure 2 using equations (1), (2), (3) and (4).

$$n_T^{\mathrm{Lit.}} = 1.000273$$
$$n_0^{\mathrm{Lit.}} = 1.000293$$

and see that the *up*-measurements are indeed more accurate.

4 Conclusion

Using an Interferometer of Jamin, we calculated the refractive index of air at ambient pressure ($(952.80 \pm 0.05)\,\mathrm{Pa}$) and at temperatures $(23.5 \pm 0.5)\,^\circ\mathrm{C}$ and $0\,^\circ\mathrm{C}$ by looking at the interference patter between a laser going through ambient pressure and one going through lower pressure. During our experiments we saw that taking measurements by increasing the pressure was easier and therefore yielded more accurate results. With our measurements we calculated the refractive index at $(23.5 \pm 0.5)\,^\circ\mathrm{C}$ and at $0\,^\circ\mathrm{C}$ to be $n_T = 1.000253 \pm 0.000007$ and $n_0 = 1.000275 \pm 0.000008$ respectively.