32 - Measuring Magnetic Fields

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April 4, 2022

Abstract

In this lab we measured a magnetic field between two poles with two different methods: Using a rotating copper coil and measuring the induced current and using cotton's scale, measuring the force acting on a current carrying wire in the magnetic field. The induction method yielded $H = (200 \pm 40)\,\mathrm{kT}$, where as the cotton's scale method gave $H = (300 \pm 4)\,\mathrm{kT}$. Finally we determined the permeability of water using the capillary method, and got $\mu_{water} = (999\,986 \pm 2) \times 10^{-6}$, which is slightly smaller, but in the same order as literature values.

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1 Introduction

A magnetic field around a conductor or between the poles of an electromagnet can be measured through induction processes, the force acting on the matter in locally variable fields, the Lorentz Force, the Biot-Savart force, or magnetic dipole moments. In vacuum, we thus get

$$\vec{B} = \mu_0 \vec{H},$$

where $\vec{B}$ is the magnetic induction, $\vec{H}$ is the magnetic field intensity, and $\mu_0 = 4\pi 10^{-7}\mathrm{VsA^{-1}m^{-1}}$ is the vacuum permeability. The magnetic flux is given as $\varphi = \int_S B_n dS$ and the induced voltage is

$$V_{ind} = -\frac{\partial \varphi}{\partial t}$$

as stated by the induction Faraday's law. If we consider an isotropic material, we get

$$\vec{B} = \mu\mu_0 \vec{H},$$

where μ is the magnetic permeability of the material and $\mu - 1$ is the magnetic susceptibility. If $\mu > 1$, we speak of a paramagnetic substance, whereas if $\mu < 1$, we have a diamagnetic substance. The susceptibility of many paramagnetic materials mainly depends on the temperature.

Consider figure 1. A small coil Sp with n loops rotates around a fixed axis a with a constant angular velocity ω. Here, a lies in the coil plane and stands perpendicular to the measured magnetic field H and the ends of the coil are tapped with sliding contacts. An induced voltage generated at the ends of the coil is given as

$$V_{ind} = A\sin(\omega t),$$

where $A = \mu_0 H n S \omega$ is the amplitude of the AC voltage. The magnetic field H can then be calculated using

$$H = \frac{A}{\mu_0 n S \omega}. \tag{1}$$

With the Biot-Savart force $F = \mu_0 I \ell H$, where ℓ is the length of the conductor piece perpendicular to the direction of H, we get for the magnetic field:

$$H = \frac{mg}{\mu_0 I \ell}. \tag{2}$$

For this, we have to use the Cotton's scale shown in figure 2. The lower part of the current loop has to be brought into the horizontal magnetic field in a way that the tangential force acts on the radial conductor piece ℓ. Then, weights $G = mg$ are added to the other side of the scale to create an equilibrium.

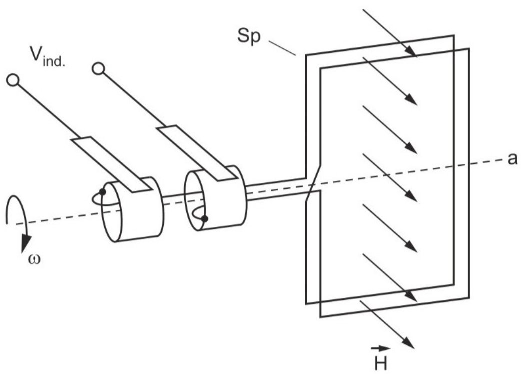

Figure 1: Small coil Sp with n whorls rotating around a fixed axis a with a constant angular velocity ω (source: [1])

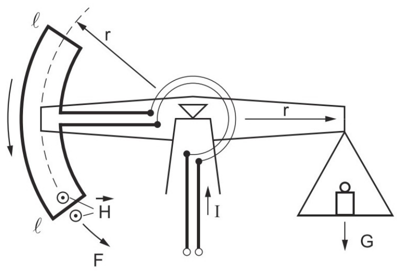

Figure 2: Cotton's scale used to determine the magnetic field with help of the Biot-Savart force (source: [1])

When placing one leg of a U-pipe (figure 3) filled with a liquid of permeability μ_F in a magnetic field, we can determine whether the liquid is paramagnetic (the liquid meniscus level rises) or diamagnetic (the liquid meniscus level is lowered). μ_F can be calculated using

$$\mu_F = \mu_L + \frac{2\rho g \delta h}{\mu_0 H^2 \mu_L^2}, \tag{3}$$

where $\mu_L = 1 + 0.38 \cdot 10^{-6}$ is the permeability of air.

2 Experiment

Part 1: Induction Method We determined the magnetic field between the poles of an electromagnet through the induced voltage in a rotating coil. To measure the amplitude and the frequency of this alternating voltage, we used an oscilloscope. The amplitude $A(\omega)$ was measured for at least five rotation rates. We then plotted it against the angular frequency ω. Next, the magnetic field was determined from the slope of $A(\omega)$.

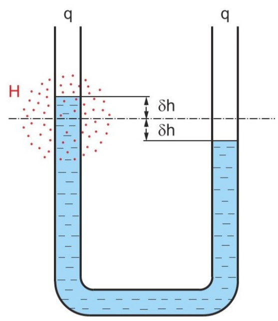

Figure 3: Displacement of a liquid in a U-pipe caused by a magnetic field H (source: [1])

Part 2: Cotton's scale We measured the force F acting on a conductor element as a function of the current I in the same field using the Cotton's scale. Then, we estimated the magnetic field. Before taking the measurement, we balanced the scale exactly at the value of the magnetic field H without I.

Part 3: Permeability of water Lastly, we measured the permeability μ of water in a known magnetic field using the setup shown in figure 4. For this, we used the capillary method.

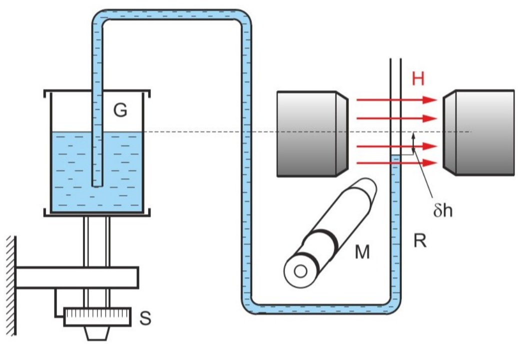

Figure 4: Setup for the determination of the permeability of water. The water level in the tube R changes by δh When the magnetic field H is switched on or off. This change in the water level can be measured by adjusting and compensating the height of a crosshair in the telescope M (source: [1])

3 Results

Part 1: Induction Method The plot depicting the amplitude A of the induced voltage in a rotating coil as a function of the angular frequency ω of the coil is shown in figure 5.

With a slope of $(2.9 \pm 0.5) \times 10^{-3}$, we got a magnetic field of

$$H = (200 \pm 40)\,\mathrm{mT}.$$

The error on the slope is estimated to be an upper bound.

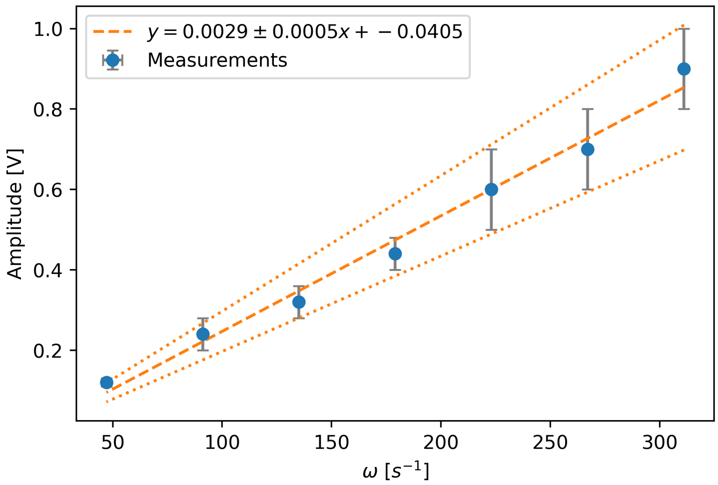

Figure 5: Amplitude measurements A against the angular frequency ω of the rotating coil used in part 1 of the experiment. The error on the linear fit is estimated to be an upper bound.

Part 2: Cotton's scale Figure 6 shows the force F acting on a conductor element as a function of the current I in the same field as before. The magnetic field obtained using the Cotton's scale method was

$$H = (300 \pm 4)\,\mathrm{kT}.$$

Part 3: Permeability of water For the permeability of water, we received a value of

$$\mu_{water} = (999\,986 \pm 2) \times 10^{-6}.$$

To obtain this value we used the average magnetic field from the previous two experiments $H = (250 \pm 20)\,\mathrm{kT}$.

4 Data analysis

We had to adjust the current I used for the electromagnet throughout the experiment so that it was always 1A. The oxidization of the copper led to a change in the magnetic field, thus creating fluctuations in I.

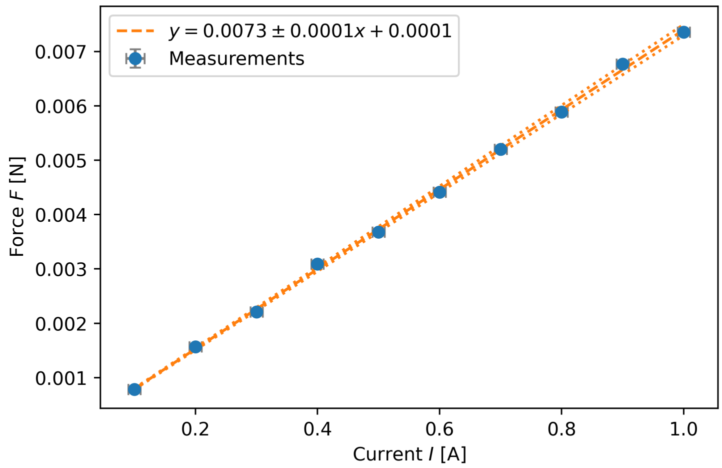

Figure 6: Force F acting on a conductor element as a function of the current I

In the first part, there could have been an error caused by inaccuracies of the oscilloscope. Since the values shown by the oscilloscope fluctuated throughout the experiment, we fixated specific moments and read off the measurements by manually adjusting the oscilloscope. This might have led to a bigger error in our value for the magnetic field.

For the Cotton's scale, we accounted for an error of ± 10mg since adding or removing 10mg of weight sometimes did not cause a disturbance of the equilibrium. The scale was rather difficult to keep at a point of rest. To minimize the error caused by this, we fixated the beam of the scale and slowly lowered it to have as little unnecessary motion of the beam as possible.

The permeability of the food coloring in the water is different to that of the water itself. This might have caused slight errors in the evaluation of the magnetic field. In order to get as precise values as possible, we took the border between the water and the air, since we had the highest contrast between colour and no colour there. Thus, it was easier to bring the water level to the correct position. For the calculation of the permeability of water, we used the average between the magnetic fields obtained in part 1 and 2. However, taking either one of the obtained values instead of their average would have led to very similar results that only differ from the fifth digit after the comma on.

Overall, we used Gaussian error propagation to get the errors on the measurement values:

$$\Delta f = \sqrt{\left(\frac{\partial f}{\partial x_1}\right)^2 \Delta x_1^2 + \cdots + \left(\frac{\partial f}{\partial x_n}\right)^2 \Delta x_n^2}. \tag{4}$$

5 Discussion

The measurements of the magnetic field with the two different methods differ by around 40%, where the induction method yields a substantially smaller value than cotton's scale.

This means that either the rotating coil was very inefficient with inducing a current, or that cotton's scale was imbalanced. An imbalance in cotton's scale would only introduce an offset in figure 6 and not change the slope. Therefore we assume that the induction method was rather inaccurate.

As for the permeability of water, given that it is a diamagnetic substance and that we got a value $\mu_{water} < 1$, we are definetly in the right order. This is further confirmed by the literature value for the permeability of water, $\mu_{water,lit.} = 0.999991$[2], which is very slightly bigger.

6 Conclusion

In conclusion we measured the magnetic field using two different methods: The induction method yielded $H = (200 \pm 40)\,\mathrm{kT}$, where as the cotton's scale method gave $H = (300 \pm 4)\,\mathrm{kT}$. We believe that the cotton's scale measurement is more accurate, as any imbalance in the scale, would only offset the graph in figure 6, and not affect the slope, from which the magnetic field is calculated. Finally, we determined the permeability of water using the capillary method, and got $\mu_{water} = (999\,986 \pm 2) \times 10^{-6}$, which is slightly smaller, but in the same order as literature values.

References

[1] https://ap.phys.ethz.ch/Anleitungen/Bilingual/32_Manual.pdf

[2] https://www.spektrum.de/lexikon/physik/permeabilitaetszahl/11059

7 Appendix

ETHZ **Physikalisches Praktikum**

Name 1: Manuel Antoinette Datum: 04.04.22

Name 2: Sujani Muguntan Platz Nr: 2

32	Messung magnetischer Felder

Magnetstrom I = 1 ± 0.05 A

1. Induktionsmethode

Rotierende Spule: n = 100, S = 1,12 cm^2

ω [1/s]	311.02	267.04	223.05	179.07	135.09	91.11	47.12
A	0.9	0.7	0.6	0.44	0.32	0.24	0.12

Grafische Darstellung: A (ω), inklusive Ursprung (ω=0, A=0).
Ausgleichsgerade (mit Fehler) berechnen.

μ_o H n S = 0.0028 ± 0.0005

H = 204'239 ± 35526 T

2. Cotton'sche Waage

I = Strom der Cotton-Waage

F = Biot-Savart-Kraft

ℓ = 1,95 cm

I [A]	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
F [N]	0.0008	0.002	0.002	0.003	0.004	0.004	0.005	0.006	0.007	0.007
H [T]	320'269	320'269	300'252	315'265	300'252	300'252	303'111	300'252	306'924	300'252

Grafische Darstellung: F (I), inklusive Ursprung (I=0, F=0).
Ausgleichsgerade (mit Fehler) berechnen.

H = 299'281 T

3. Permeabilität des Wassers

δ h [m]	$-4.05 \cdot 10^{-5}$	$-5.60 \cdot 10^{-5}$	$-6.70 \cdot 10^{-5}$	$-5.80 \cdot 10^{-5}$	$-6.40 \cdot 10^{-5}$	$-5.80 \cdot 10^{-5}$

< δ h > = $-5.72 \cdot 10^{-5}$ m

$\mu_{Luft} = 1 + 0,38 \cdot 10^{-6}$

μ_{H_2O} = 0.999986

24.09.2021