Interference and Diffraction

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Abstract

In this lab, we measured the wavelength $\lambda = (540 \pm 70)\,\text{nm}$ of a green laser using a Fresnel bipsirm, by measuring the distance between maxima in the focal plane, and the angle between the wave fronts. Next we measured and calculated the distance between maxima in the focal plane for a wire instead of a Fresnel biprism yielding $d_{\text{meas.}} = (6.3 \pm 0.5)\,\text{mm}$ and $d_{\text{theor.}} = (6 \pm 3)\,\text{mm}$. Then we replaced the wire with a slit and measured the width of the slit $b = (0.083 \pm 0.004)\,\text{mm}$. We also measured the diameter of lykopodium particles using a microscope $D_{\text{meas.}} = (0.038 \pm 0.004)\,\text{mm}$ and by observing the diffraction pattern it creates $D_{\text{exp}} = (0.017 \pm 0.002)\,\text{mm}$. Lastly, we measure the thickness of a strand of hair $b = (0.07 \pm 0.02)\,\text{mm}$.

1 Introduction

1.1 Interference Effects at a Fresnel Biprism

The Fresnel biprism consists of a triangular prism with a dihedral angle of almost 180 degrees. If the parallel light beam of an infinitely distant slit S fall on these surfaces, it is split by the biprism into two bundles 1 and 2, which interfere behind the prism. We replicate the effects on the infinitely distant slit by setting up the slit S at a distance of focal length f_1 in front of the lens L. We can use an eyepiece O to see the interference pattern generated in the focal plane F. At an arbitrary point P in this plane, the phase difference between the two waves can be calculated with

$$\delta = a\frac{2\pi}{\lambda} - (-a\frac{2\pi}{\lambda}),$$

where $a = x \sin\frac{\beta}{2}$, β is the angle between both waves, and x is the distance of our point P from the center of the of eyepiece. Since the phase difference of two maxima is $\delta = p2\pi$, with $p \in \text{N}$, it follows that

$$x = \frac{p\lambda}{2\sin\frac{\beta}{2}}.$$

The distance between two neighboring maxima is $2x$.

1.2 Diffraction at a slit and a thin wire

Here we place the source Q in the focal plane f_1 of a convergent lens L_1 and image the diffraction pattern with a second lens L_2. We then place a screen S parallel to and between L_1 and L_2, in order to create a slit with the width b. The intensity distribution of the diffraction pattern behind the slit at an arbitrary point P as a function of the distance r_0 to the lower slit border and the angle φ can be found with

$$I(\varphi) = \frac{A^2 b^2}{r_0^2} \frac{\sin^2(\pi \frac{b \sin\varphi}{\lambda})}{(\pi \frac{b \sin\varphi}{\lambda})^2},$$

where λ is the wavelength of the light and A^2 the intensity of the light incident on the slit. Intensity minima occur when $p\lambda = b\sin\varphi_p$ and maxima when $(p+1/2)\lambda = b\sin\varphi_p$. If we replace the screen with a wire, the diffraction pattern will be identical, except for $\varphi = 0$, according to Babinet's principle.

The diffraction pattern of m randomly distributed diffraction centers, which are geometrically identical, have the same diffraction pattern as the one of one single center, with the difference that intensity is m times bigger. The diffraction pattern of a circular aperture is given by

$$I(\varphi) = \frac{4A^2}{r_0^2} \frac{J_1^2(\pi \frac{D \sin\varphi}{\lambda})}{\pi \frac{D \sin\varphi}{\lambda})^2},$$

where D is the diameter of the aperture. We find the first minimum circle for $1.22\lambda = D\sin\varphi$.

In the first experiment we have to determine the wavelength λ of the diode laser using the fresnel biprism. We will do this by measuring the distance between two maxima d' on a screen a distance b from a lens L_1 and using the equation $d = d'\frac{a}{b}$, where d is then the actual distance of the maxima and a the distance from the lens to the plane R in front of the biprism. We can determine the angle β by using a setup with two lenses, measuring the distance between two maxima l using the same method as before, and using the equation $\beta \approx \frac{l}{L-A}$.

Second, we will be determining the distance between two neighboring maxima by replacing the biprism with a wire with the equation$d = \frac{\lambda}{b}S$, where S is the distance between the wire and the screen and b the thickness of the wire. Using a slit instead of a wire, we have the equation $b = b'\frac{A}{B}\frac{C}{L-A}$, where b' is the distance we measure, $L = L_2 - L_1$, C is the distance between the slit and L_2.

Third we will determine the diameter of Lykopodium particles from the diffraction pattern they generate. The radius of minimum intensity is given by $\rho_1 = 1.22L\frac{\lambda}{D}$, where L is the distance between the glass plate and the screen and D is the diameter of the particles.

2 Experiment and Results

2.1 Interference effects at a Fresnel Biprism

For the first part of the experiment, we had to determine the wavelength λ of the diode laser using the fresnel biprism. We did this by measuring the distance between two maxima and the angle between two maxima and using equation (4). The set up consisted of a laser, the Fresnel biprism, lenses L_1, L_2 and a screen P. We measured the distance between two neighboring maxima d' on a screen a distance $b = (54.4 \pm 0.2)\,\text{cm}$ magnified by the lens L_1 with focal length $f_1 = (11.0 \pm 0.5)\,\text{mm}$ and using the equation

$$d = d'\frac{a}{b}, \tag{1}$$

where d is the actual distance between the maxima and $a = (1.23 \pm 0.50)\,\text{cm}$ the distance from the lens to the plane R in front of the biprism, which we calculated using the relation

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f}. \tag{2}$$

We measured a distance $d' = (4.6 \pm 0.5)\,\text{mm}$, between the two maxima on the screen P. This yielded a distance $d = (0.09 \pm 1.00)\,\text{mm}$ between the maxima in the focal plane R.

Then we determined the angle β, at which the wave fronts are tilted after leaving the prism, by using a setup with two lenses, L_2 with focal length $f_2 = (10.0 \pm 0.5)\,\text{cm}$ and L_1 from before. The first lens groups the wave fronts into two points, and the second lens projects the image onto the screen. By fixing L_2 and P and adjusting the position of L_1, we were able to display two dots on the screen. We measured the distance between the two maxima on the screen $l' = (0.90 \pm 0.05)\,\text{cm}$, the distance $B = (16.6 \pm 0.2)\,\text{cm}$ between L_1 and P, and the distance $L = (11.6 \pm 0.2)\,\text{cm}$. We were able to calculate the distance $A = (1.18 \pm 0.06)\,\text{cm}$ between lens L_1 and the focal plane R using equation (2), and then with equation (1) we could calculate the actual distance $l = (0.64 \pm 0.03)\,\text{mm}$ between the two maxima in the focal plane. Then, using

$$\beta \approx \frac{l}{L - A}, \tag{3}$$

we were able to calculate the angle $\beta = (0.0060 \pm 0.0003)\,\text{rad}$. Using the results, we calculated the wavelength of the diode with the equation

$$\lambda = 2d \sin\left(\frac{\beta}{2}\right), \tag{4}$$

which yielded the result $\lambda = (540 \pm 70)\,\text{nm}$.

2.2 Diffraction at a Slit and Wire

For the next set of experiments, the setup consisted of the laser, either a wire or a slit, and the screen P. First we measured the distance between maxima in the diffraction pattern of a wire with thickness $b = (0.10 \pm 0.05)\,\text{mm}$. The wire was placed at a distance $S = (120.0 \pm 0.2)\,\text{cm}$ from the screen P. We measured a distance $d_{\text{meas.}} = (6.3 \pm 0.5)\,\text{mm}$. We also calculated the theoretical value using the previously measured wavelength $\lambda = (540 \pm 70)\,\text{nm}$ with the equation

$$d_{\text{theor.}} = \frac{\lambda}{b} S, \tag{5}$$

for which we got $d_{\text{theor.}} = (6 \pm 3)\,\text{mm}$.
Then we replaced the wire with a slit, placed at the same distance S from the screen as the wire was. We placed the lens L_2 a distance $C = (14.9 \pm 0.2)\,\text{cm}$ in front of the slit, and the lens L_1 a distance $L = (27.1 \pm 0.2)\,\text{cm}$ from L_2 and $B = (77.2 \pm 0.2)\,\text{cm}$ from the screen. We measured a distance $\tilde{b} = (10.0 \pm 0.5)\,\text{mm}$ on the screen. With equation (2) we calculated the distance $A = (1.1 \pm 0.5)\,\text{cm}$ between the focal plane R and L_1 and then using equation (1) we calculated the length $b' = (0.145 \pm 0.007)\,\text{mm}$ in the focal plane. We can calculate the width of the slit using the equation

$$b = b' \frac{C}{L - A}, \tag{6}$$

where we get $b = (0.083 \pm 0.004)\,\text{mm}$.

2.3 Diffraction Effects at Lykopodium Platelets

For the next experiment, we measured the diameter D of lykopodium particles. We did this by observing the diffraction pattern of these lykopodium particles. We placed a holder containing the particles about 5 cm from the laser and a distance $L = (119.2 \pm 0.2)\,\text{cm}$ from the screen. We measured the radius $\rho_1 = (4.74 \pm 0.01)\,\text{cm}$ of the minimum intensity, which obeys the following formula

$$D = 1.22 L \frac{\lambda}{\rho_1}, \tag{7}$$

where we have the wavelength λ from the first experiment. We measured the diameter to be $D_{\text{exp}} = (0.017 \pm 0.002)\,\text{mm}$. We also measured the diameter of a particle using a microscope. We measured the diameter of a particle using a microscope yielding $D_{\text{meas.}} = (0.038 \pm 0.004)\,\text{mm}$ as can be seen in Figure 1. The scale for the microscope was $10 = 0.38\,\text{mm}$

2.4 Measurement of the Pin-size Hole

Next, we measured the diameter of a pin-size hole using the equation

$$D = pL \frac{\lambda}{\rho_p}, \tag{8}$$

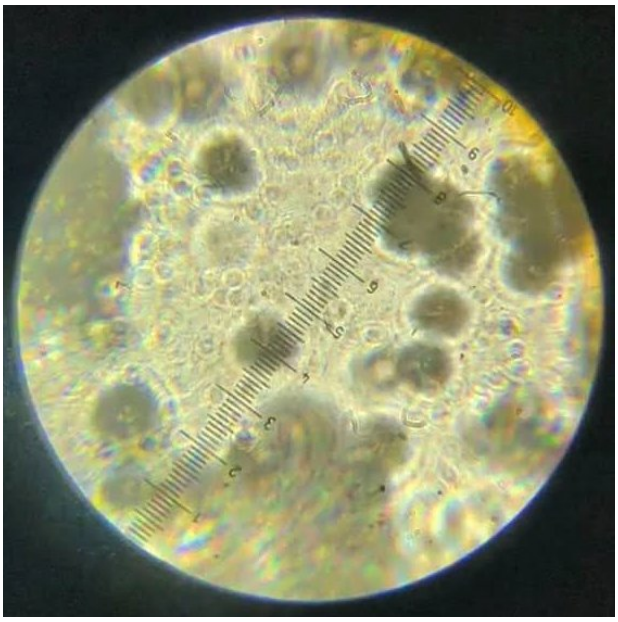

Figure 1: Lykopodium Platelets under the microscope. One unit on the scale is 0.038 mm.

where p is the order of the minimum, ρ_p the corresponding radius, D the diameter of the hole, $L = (119.2 \pm 0.2)$ cm, and λ the wavelength of the laser. We measured $\rho_1 = (5.00 \pm 0.05)$ mm and $\rho_2 = (10.00 \pm 0.05)$ mm, which both led to the result $D = (1.3 \pm 0.2)$ µm.

2.5 Thickness of a Strand of Hair

Lastly, we measured the thickness of a strand of hair. We held the hair a distance of about $L = (142.0 \pm 0.2)$ cm from the screen. We measured a distance $d' = (10.20 \pm 0.05)$ mm between maxima. Using equation (5), we calculated the thickness $b = (0.07 \pm 0.02)$ mm.

3 Data Analysis

We calculated the error of the distance between the focal plane and the lens R, which is the error for equation (2) using the Gauss propagation method. The equation is

$$\Delta a = \sqrt{\left(\frac{b(b-f)+bf}{(b-f)^2}\right)^2 (\Delta f)^2 + \left(\frac{f(b-f)-bf}{(b-f)^2}\right)^2 (\Delta b)^2}.$$

We calculated the error of the distance between maxima in the focal plane R in experiment one, the error for equation (1), using the Gauss propagation method. The equation is

$$\Delta d = \sqrt{\left(\frac{a}{b}\right)^2 (\Delta d')^2 + \left(\frac{d'}{b}\right)^2 (\Delta a)^2 + \left(\frac{d'a}{b^2}\right)^2 (\Delta b)^2}.$$

We used the same method to calculate the error of β in equation (3). The equation we used is

$$\Delta\beta = \sqrt{\left(\frac{1}{L-A}\right)^2 (\Delta l)^2 + \left(\frac{l}{(L-A)^2}\right)^2 (\Delta L)^2 + \left(\frac{l}{(L-A)^2}\right)^2 (\Delta A)^2}.$$

With Gauss, the error for the wavelength in equation (4) is calculated by

$$\Delta\lambda = \sqrt{\left(2\sin\left(\frac{\beta}{2}\right)\right)^2 (\Delta d)^2 + (d\cos\left(\frac{\beta}{2}\right))^2 (\Delta\beta)^2}.$$

The error for $d_{\text{theor.}}$ in equation (5) is calculated with the equation

$$\Delta d_{\text{theor.}} = \sqrt{\left(\frac{S}{b}\right)^2 (\Delta\lambda)^2 + \left(\frac{\lambda}{b}\right)^2 (\Delta S)^2 + \left(\frac{\lambda S}{b^2}\right)^2 (\Delta b)^2}.$$

The error for the width of the slit in equation (6) was calculated with the equation

$$\Delta b = \sqrt{\left(\frac{C}{L-A}\right)^2 (\Delta b')^2 + \left(\frac{b'}{L-A}\right)^2 (\Delta C')^2 + \left(\frac{b'C}{(L-A)^2}\right)^2 (\Delta L)^2 + \left(\frac{b'C}{(L-A)^2}\right)^2 (\Delta A)^2}.$$

The error for the diameter of a lykopodium particle in equation (7) is calculated with

$$\Delta D = 1.22\sqrt{\left(\frac{\lambda}{\rho_1}\right)^2 (\Delta L)^2 + \left(\frac{L}{\rho_1}\right)^2 (\Delta\lambda)^2 + \left(\frac{\lambda L}{\rho_1^2}\right)^2 (\Delta\rho_1)^2}.$$

The error for the diameter of a pin-size hole in equation (8) was calculated with

$$\Delta D = p\sqrt{\left(\frac{\lambda}{\rho_p}\right)^2 (\Delta L)^2 + \left(\frac{L}{\rho_p}\right)^2 (\Delta\lambda)^2 + \left(\frac{\lambda L}{\rho_p^2}\right)^2 (\Delta\rho_p)^2}.$$

4 Discussion

In the first experiment, we measured a wavelength $\lambda = (540 \pm 70)\,\text{nm}$, which lies within the range 520-570 nm for a green laser. In the second experiment, the theoretical $d_{\text{theor.}} = (6 \pm 3)\,\text{mm}$ and measured $d_{\text{meas.}} = (6.3 \pm 0.5)\,\text{mm}$ values for the distance between maxima of the diffraction pattern at a wire lie within the real of uncertainty to each other and are therefore very close. The measured diameter D_{exp} of a lykopodium particle using the diffraction pattern was $(0.017 \pm 0.002)\,\text{mm}$ opposed to the size of the diameter we measured using the microscope $D_{\text{meas}} = (0.038 \pm 0.004)\,\text{mm}$. Taking the error measurements into consideration and the fact that lykopodium particles vary in size, we consider this to be an accurate measurement. Lastly we measured the thickness of a strand of hair to be $b = (0.07 \pm 0.02)\,\text{mm}$. Hair thickness ranges from 0.06 to 0.1 mm according to this link, so our measurement makes sense.

5 Conclusion

In conclusion, we are pleased with the measurements made in each experiment, since they seem to be accurate and fall in line with our expectations.