Hall Effect

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Abstract

We calculated the conductivity of copper, silver, and aluminum yielding the results: $\sigma_{\text{exp,Cu}} = (4.6 \pm 0.7) \times 10^7\,\Omega^{-1}\,\text{m}^{-1}$, $\sigma_{\text{exp,Ag}} = (6 \pm 1) \times 10^7\,\Omega^{-1}\,\text{m}^{-1}$, and $\sigma_{\text{exp,Al}} = (3.3 \pm 0.6) \times 10^7\,\Omega^{-1}\,\text{m}^{-1}$. We measured the Hall voltage for each metal placed in a magnetic field, once varying the current while keeping the magnetic field constant and vice versa. We calculated the Hall coefficient for each metal to be $R_H^{\text{exp,Cu}} = (-6 \pm 2) \times 10^{-11}\,\Omega\,\text{m}\,\text{T}^{-1}$, $R_H^{\text{exp,Ag}} = (-9 \pm 2) \times 10^{-11}\,\Omega\,\text{m}\,\text{T}^{-1}$, and $R_H^{\text{exp,Al}} = (-3.4 \pm 0.8) \times 10^{-11}\,\Omega\,\text{m}\,\text{T}^{-1}$.

1 Introduction

According to Drude's theory of electrical conductivity, the conduction electrons in a metal behave like free electrons. To describe the velocity of the conduction electrons in the metal, while an electric field $\vec{E}$ is applied, we must also examine the interactions of the electrons, which consist of collisions between the electrons to the ions and between electrons with other electrons. For this we introduce a mean scattering time τ, which is defined as the time that elapses on average between two collisions. The drift velocity caused by $\vec{E}$ is given by

$$v_D = \frac{-e}{m_e} E t,$$

where t is the time elapsed since the last collision, e is the charge of an electron, and m_e is the mass of the electron. The relation to the current density j and the electrical conductivity of the metal σ are:

$$j = -ne\langle v_D \rangle = \tfrac{ne^2\tau}{m_e} E \text{ and } \sigma = j/E = \tfrac{ne^2\tau}{m_e}$$

The theory of the Hall Effect says, that if a current of density $\vec{j}$ flows through a rectangular metal plate in the positive x-direction and through a magnetic field with flux density $\vec{B}$ which flows in the z-direction, then there will be a small voltage V_H between the boundary surfaces perpendicular to y-direction. This is caused by the Lorentz force $\vec{F}_L = -e(\vec{E} +$

$\vec{v} \times \vec{B}$). From this equation we get

$$\vec{a} = \frac{-e}{m}(\vec{E} + \vec{v} \times \vec{B}).$$

Assuming Drude's theory is correct, we can obtain the components of the drift velocity with:

$v_x = \frac{-e\tau}{m}(E_x + v_y B_z)$,

$v_y = \frac{-e\tau}{m}(E_y + v_x B_z)$, and

$v_z = 0$,

which correspond to the current densities $j_x = -env_x$ and $j_y = -env_y$. If we measure the Hall voltage in a static way with $v_y = 0$, then we get the equation $E_y = -\frac{1}{en} j_x B_z = -\frac{1}{en}\sigma E_x B_z$. The material specific Hall coefficient is then

$$R_H = \frac{E_y}{j_x B_z} = -\frac{1}{en}, \tag{1}$$

$$R_H = \frac{E_y}{j_x B_z} = \frac{V_H l_z}{I_x B_z}. \tag{2}$$

The Hall angle is defined as $\theta \approx tan(\theta) = E_y/E_x = R_H \sigma B_z$ and is the angle between the original electric field strength flowing through the magnetic field and the strength of the resulting external electric field. The Hall mobility is defined as

$$b_H = |R_H \sigma|,$$

which leads to $\theta = b_H B_z$, since both R_H and σ are negative for a metal.
In this lab, we will measure the conductivity of 3 different metals; copper, silver, and aluminum, by sending a DC current I through the x-axis of a rectangular cuboid made out of the metal and then measuring the voltage drop V_x between two points on the x-axis. The equation $\sigma = \frac{I_x l_x}{V_x l_y l_z}$ will then allow us to calculate the specific conductivity of the metal.
We will then measure the Hall voltage by placing the same setup we used the the measurement of the conductivity into a magnetic field created by two solenoids. In the first experiment we will keep the magnetic field constant $B = 1\,\mathrm{T}$, while measuring the Hall voltage for different currents I_x between 0 and 12A and in the second experiment we will keep the current constant $I_x = 3\,\mathrm{A}$, while varying the magnetic field B between -1 and 1T. Using a lock in amplifier, we will be able to measure the Hall voltage without any imposing signals from the external induced interference voltages in the signal lines.
With the equation (2) we can determine the Hall coefficient from our measured values. With (1) we can calculate the effective density n_{exp} from the experimental value we get for R_H. With this we will be able to calculate the scattering time τ_{exp} with the equation

$$\tau_{\text{exp}} = \frac{\sigma_{\text{exp}} m_e}{n_{\text{exp}} e^2}$$

and our measurement for σ_{exp} from the first experiment.

2 Experiment and Results

2.1 Conductivity of Metal Foils

Metal	l_x [mm]	l_y [mm]	l_z [mm]	V_x [mV]	σ_{exp} $[10^7\Omega^{-1}\text{m}^{-1}]$	$\sigma_{\text{Lit.}}$ $[10^7\Omega^{-1}\text{m}^{-1}]$
Cu	49.7	11.0	0.035	14.0	4.6 ± 0.7	5.96
Ag	49.7	11.0	0.025	15.0	6 ± 1	6.3
Al	49.7	11.0	0.030	22.8	3.3 ± 0.6	3.5

Table 1: The Dimensions and conductivity of the three metals used in the experiment and the measured voltage drop, when sending a 5 A current along the x-axis. The corresponding errors for the dimensions is half of the last significant digit (e.g. $l_{x,\text{Cu}} = (49.70 \pm 0.05)\,\text{mm}$). The literature values were taken from https://sciencenotes.org/table-of-electrical-resistivity-and-conductivity/.

Our first task in this lab was to measure the conductivity of copper, silver and aluminum. The dimensions of the metals can be seen in table 1. We sent a DC-current of $I_x = (5.00 \pm 0.05)\,\text{A}$ through the x-axis of the metal plates and measured the voltage drop V_x along the x-axis. Using

$$\sigma = \frac{I_x l_x}{V_x l_y l_z}$$

we can calculate the conductivity of each metal which can be found in table 1.

2.2 Hall Voltage

The main part of this lab was measuring the induced Hall voltage that results, when we place the setup from the measurement of conductivity inside of a magnetic field. We sent an AC-current through the x-axis of each metal, and measured the Hall voltage that occurs in the y-axis. The voltage drop we measure along the y axis has both an x and a y component. We only want the y-component, which is the Hall voltage, so we used a potentiometer in order to calibrate the measurement and minimize the x-component of the measurement. This is crucial to the experiment, because I_x is several orders of magnitude greater than I_y.

We measured the voltage drop using a lock-in amplifier, in order to minimize the noise signal which can affect our measurement of the voltage drop.

Our magnetic field was generated by an electromagnet with a small air gap in the middle in order to place the metal inside of the magnetic field. The magnetic field strength depends on the current going through the coils, but because of the characteristics of the electromagnet, we first had to raise the current I_B going through the coils to a minimum of 3.3 A, in order to fully activate the electromagnet. Then we could set the magnetic field strength by setting the current to the value in the table given to us, for example for $B = 0\,\text{T}$ we set a current of $I_B \approx -0.07\,\text{A}$.

	Cu	Ag	Al	Unit
R_H^{exp}	-6 ± 2	-9 ± 2	-3.4 ± 0.8	$10^{-11}\Omega\,\text{m}\,\text{T}^{-1}$
$1/(n_{\text{Lit.}}e)$	-7.3	-1.1	-3.5	$10^{-11}\text{m}^3/\text{C}$
$R_H^{\text{Lit.}}$	-5.4	-9.0	-3.3	$10^{-11}\Omega\,\text{m}\,\text{T}^{-1}$
b_H^{exp}	2.9 ± 0.7	5 ± 1	1.1 ± 0.3	10^{-3}T^{-1}
$n_{\text{Lit.}}$	11.8	8.3	24	10^{28}m^{-3}
n_{exp}	10 ± 3	7 ± 2	19 ± 5	10^{28}m^{-3}
τ_{exp}	1.6 ± 4	3.0 ± 0.7	0.6 ± 0.2	10^{-14}s

Table 2: Here is the average of the measured Hall coefficient R_H^{exp} and the Hall mobility b_H^{exp}, the effective density n_{exp}, and the scattering time τ_{exp} calculated with R_H^{exp} and σ_{exp}. The values for $n_{\text{Lit.}}$ and $R_H^{\text{Lit.}}$ were provided by the lab.

First, in experiment (a), we measured the Hall voltage V_H for $B = (1.00 \pm 0.05)\,\text{T}$, while varying I between 0 and 12A, as can be seen in figure 1.

Then, in experiment (b) we measured the Hall voltage V_H for $I = (3.0 \pm 0.1)\,\text{A}$ while varying the magnetic field B between -1 and $1\,\text{T}$ at intervals of approximately $0.2\,\text{T}$, as can be seen in figure 2.

Our last task was to calculate the Hall coefficient. For the two experiments (a) and (b) we can calculate the Hall coefficient as follows:

$$R_H^{\text{exp,a}} = s_a \frac{l_z}{B_z},$$

$$R_H^{\text{exp,b}} = s_b \frac{l_z}{I_x},$$

where $s_a = V_H/I_x$ and $s_b = V_H/B_z$ are the slopes of the linear regressions in experiment (a) and (b) respectively. Then we calculated the effective density $n_{\text{exp}} = -\frac{1}{R_H^{\text{exp}} e}$ using $R_H^{\text{exp}} = \frac{1}{2}(R_H^{\text{exp,a}} + R_H^{\text{exp,b}})$ and the scattering time $\tau = \frac{\sigma^{\text{exp}} m}{n_{\text{exp}} e^2}$ with the results and measurements from before. All these results can be found in table 2.

3 Data Analysis

Using the Gauss error propagation method, we can calculate the error of our measurement of the conductivity of each metal $\Delta\sigma$ with the following equation:

$$\Delta\sigma = \sqrt{\left(\frac{l_x}{V_x l_y l_z}\right)^2 \Delta I_x^2 + \left(\frac{I_x l_x}{V_x^2 l_y l_z}\right)^2 \Delta V_x^2 + \left(\frac{I_x}{V_x l_y l_z}\right)^2 \Delta l_x^2 + \left(\frac{I_x l_x}{V_x l_y^2 l_z}\right)^2 \Delta l_y^2 + \left(\frac{I_x l_x}{V_x l_y l_z^2}\right)^2 \Delta l_z^2}.$$

We can calculate the associated error of the material specific Hall coefficient for the two experiments (a) and (b) with the equations

$$\Delta R_H^{\text{exp,a}} = \sqrt{\left(\frac{l_z}{B_z}\right)^2 \Delta s_a^2 + \left(\frac{s_a l_z}{B_z^2}\right)^2 \Delta B_z^2 + \left(\frac{s_a}{B_z}\right)^2 \Delta l_z^2},$$

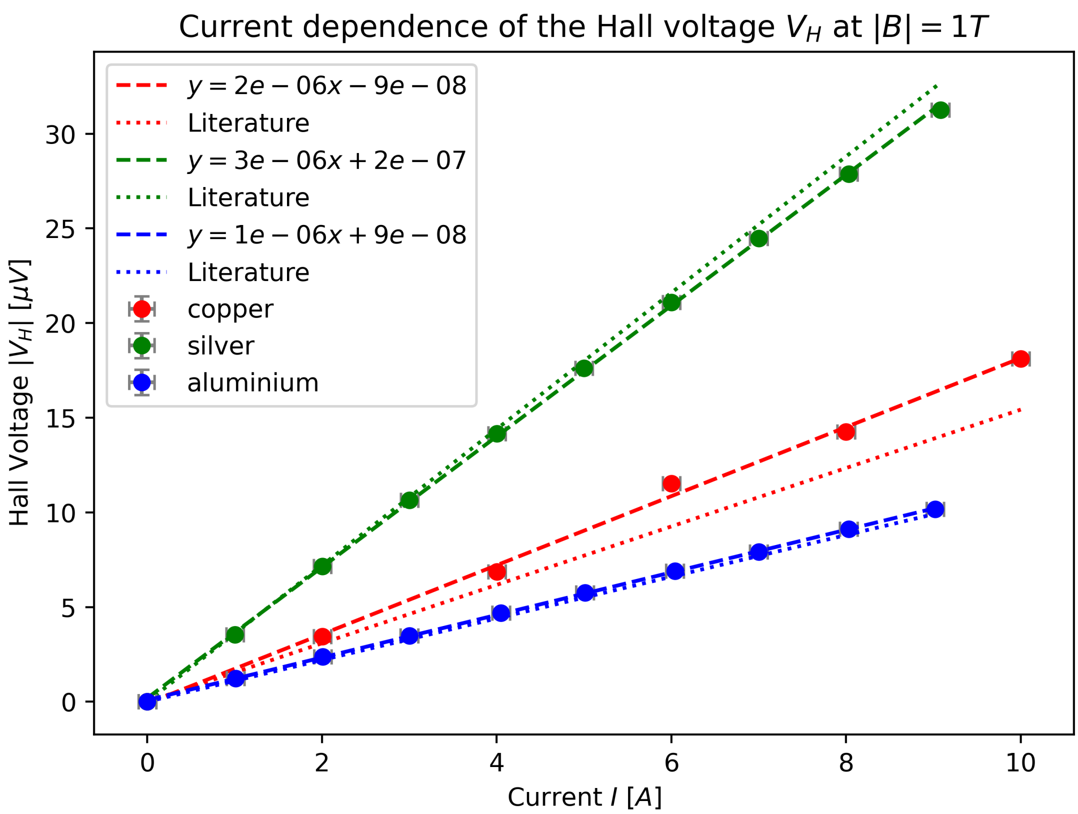

Figure 1: The figure depicts the results from experiment (a): The current dependence of the Hall voltage for our measured values (circles), a corresponding linear regression (dashed) and the expected graph with the slope $s_a^{\text{Lit.}} = R_H^{\text{Lit.}} B_z / l_z$. The literature value $R_H^{\text{Lit.}}$ was provided by the lab.

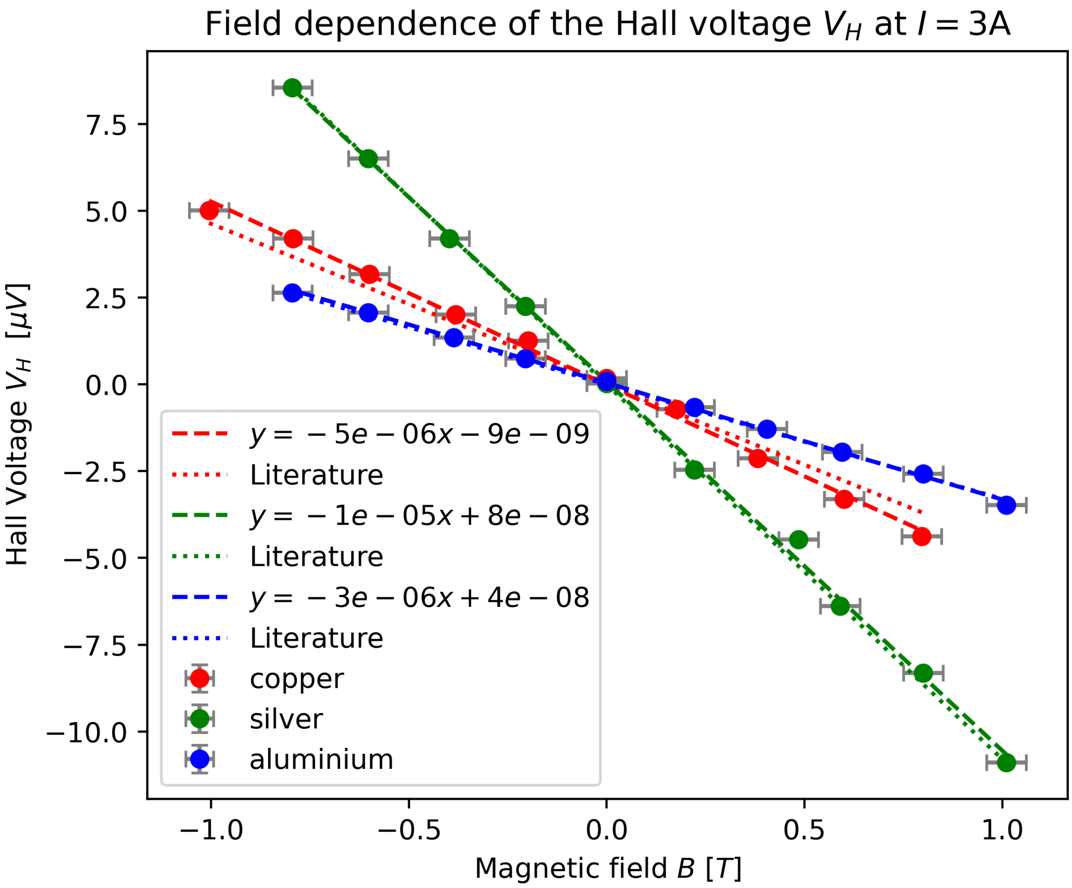

Figure 2: The figure depicts the results from experiment (b): The field dependence of the Hall voltage for our measured values (circles), a corresponding linear regression (dashed) and the expected graph with the slope $s_b^{\text{Lit.}} = R_H^{\text{Lit.}} I_x / l_z$. The literature value $R_H^{\text{Lit.}}$ was provided by the lab.

$$\Delta R_H^{\mathrm{exp,b}} = \sqrt{\left(\frac{l_z}{I_x}\right)^2 \Delta s_b^2 + \left(\frac{s_b l_z}{I_x^2}\right)^2 \Delta I_x^2 + \left(\frac{s_b}{I_x}\right)^2 \Delta l_z^2},$$

where the mean standard error of the slopes $\Delta s_{a,b}$ are given by

$$\Delta s_a = \frac{\sum_i^n \left(V_{H_i}^a - s_a I_{x_i}\right)^2}{n_a},$$

$$\Delta s_b = \frac{\sum_i^n \left(V_{H_i}^b - s_b B_{z_i}\right)^2}{n_b},$$

where n_a and n_b are the number of measurements we made. The error of the average of $R_H^{\mathrm{exp,a}}$ and $R_H^{\mathrm{exp,b}}$ is given by

$$\Delta R_H^{\mathrm{exp}} = \sqrt{\left(\frac{1}{2}\right)^2 (\Delta R_H^{\mathrm{exp,a}})^2 + \left(\frac{1}{2}\right)^2 \left(\Delta R_H^{\mathrm{exp,b}}\right)^2}.$$

We calculated the error of the number of electrons Δn_{exp} with

$$\Delta n_{\mathrm{exp}} = \frac{\Delta R_H^{\mathrm{exp}}}{(R_H^{\mathrm{exp}})^2 e},$$

and the error of the scattering time $\Delta\tau$ with the equation

$$\Delta\tau = \sqrt{\left(\frac{m}{ne^2}\right)^2 \Delta\sigma^2 + \left(\frac{\sigma m}{n^2 e^2}\right)^2 \Delta n^2}.$$

4 Discussion

The measurements of the conductivity for each metal were fairly accurate. The difference between the measured conductivity values and the literature values for silver $\Delta\sigma_{\mathrm{Ag}} = 0.3 \times 10^7\,\Omega^{-1}\,\mathrm{m}^{-1}$ and aluminum $\Delta\sigma_{\mathrm{Al}} = 0.2 \times 10^7\,\Omega^{-1}\,\mathrm{m}^{-1}$ both lie within the realm of uncertainty for each measurement $\pm 1 \times 10^7\,\Omega^{-1}\,\mathrm{m}^{-1}$ and $\pm 0.6 \times 10^7\,\Omega^{-1}\,\mathrm{m}^{-1}$ respectively. The difference between the values for copper is $\Delta\sigma_{\mathrm{Cu}} = 1.36 \times 10^7\,\Omega^{-1}\,\mathrm{m}^{-1}$, and although it does not lie within range of the error $0.7 \times 10^7\,\Omega^{-1}\,\mathrm{m}^{-1}$, the ratio between them is less than 2, so the values are relatively close.
We were also successful with our measurements of the Hall voltage. The results from both experiment returned the same value, which is equal to the average as well in table 2, with regards to the uncertainty in the measurements. The difference between the average value of the Hall coefficients and the literature value for copper and aluminum $\Delta R_H^{\mathrm{exp,Cu}} = 0.6 \times 10^{-11}\,\Omega\,\mathrm{m}\,\mathrm{T}^{-1}$ and $\Delta R_H^{\mathrm{exp,Al}} = 0.1 \times 10^{-11}\,\Omega\,\mathrm{m}\,\mathrm{T}^{-1}$ and are within the realm of uncertainty $2 \times 10^{-11}\,\Omega\,\mathrm{m}\,\mathrm{T}^{-1}$ and $0.8 \times 10^{-11}\,\Omega\,\mathrm{m}\,\mathrm{T}^{-1}$ respectively. The difference between the measured value and literature for silver is 0 with the measurement error

$2 \times 10^{-11}\,\Omega\,\mathrm{m}\,\mathrm{T}^{-1}$ for silver. Given that the measured values for the Hall coefficient and the conductivity of each metal were fairly accurate, it follows that the measured values for the effective density n_{exp}, the Hall mobility b_H^{exp}, and the scattering time τ_{exp} will be too, because they are calculated with R_H^{exp} and σ_{exp}. Indeed we can see this when comparing the measured values of the effective density to the literature values. The difference between the two lies within the measurement error for each metal as can be seen in table 2.

5 Conclusion

In conclusion, the experiments performed delivered results close to the literature values. This confirms the validity of the conductivity equation, Drude's theory, and the Hall effect.