Gyroscope

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1 Abstract

In this lab, we set out to deepen our understanding of the physics behind the movement of a gyroscope. We observed the case of a rapidly rotating and rotational symmetric gyroscope and conducted three experiments in order to calculate both the moment of inertia and the angular velocity of the precession of the gyroscope.

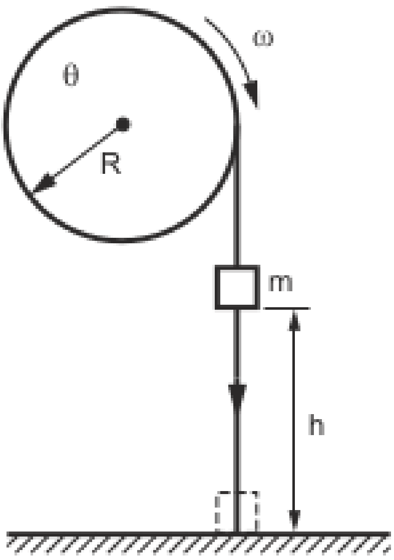

Figure 1: A depiction of what the first experiment looked like

2 Experiment

2.1 Determining the Moment of Inertia

In the first experiment, to determine the moment of inertia, we mounted a wheel to a stand with horizontal axis. We placed a rope around the wheel, with one end attached to the wheel and the other to a mass m. We then dropped the mass from a height h, increasing the angular velocity of the wheel.

The moment of inertia can be calculated using the laws of energy conservation. The sum of the potential and kinetic energy of the mass at the beginning and the end are related by the following equation

$$mgh = \frac{mv^2}{2} + \frac{\theta\omega_0^2}{2}$$

where θ is the moment of inertia. Using the fact that the velocity of the mass at the end is equal to

$$v = R\omega_0$$

leads us to the final equation

$$\theta = \frac{2mgh}{\omega_0^2} - mR^2$$

To calculate the angular velocity ω_0, we measured the period T_0 of the wheels rotation after dropping the mass m, and then used the following equation

$$\omega_0 = \frac{2\pi}{T_0}$$

2.2 Alternative way to determine the Moment of Inertia

In our second experiment, we took a different approach to determining the moment of inertia, this time measuring the time it takes for a mass m to fall a certain height h. We once again use the equation

$$mgh = \frac{mv^2}{2} + \frac{\theta\omega_0^2}{2}$$

except this time we write the velocity of the mass in terms of the acceleration at the time t:

$$v_t = a_t t$$

as well as the angular velocity:

$$\omega_t = \frac{a_t}{R}$$

Using this, we can rewrite the equation above as:

$$\theta = mR^2(\frac{gt^2}{2h} - 1)$$

2.3 Determining the Precession Velocity

In this experiment, we placed the gyroscope on the pivot bearing of the stand. We then attached a mass m on the opposing side of the gyroscope a distance l from the pivot bearing and applied a strong angular momentum to the wheel. We simultaneously measured the period of the wheel T_ω as well as the period of the gyroscope around the pivot bearing T_Ω. Using the moment of inertia calculated from the previous experiments and the fact that the angular momentum of the wheel is given by $\omega = \frac{2\pi}{T_\omega}$ and the angular velocity of the precession $\Omega_{Prec} = \frac{2\pi}{T_\Omega}$ we can check the validity of the equation:

$$\Omega_{Prec} = \frac{Gl}{\theta\omega}$$

where $G = mg$.

3 Results

3.1 First Experiment

In the first experiment, we conducted the experiment with 5 different variables. We kept the mass constant, while varying the height from which we dropped it, as can be seen in Figure 2. Then we kept the height constant, while changing the mass, which we dropped, which can be seen in Figure 3. We measured the radius of the wheel to be $R = 0.32m$. We wanted to avoid as much error as possible while measuring the period, so we measured the time it took for 5 periods and then took the average. We measured the height with a ruler, so we were able to get accurate results.

Mass [kg]	Height [m]	Average Period [s]	Angular Velocity [s^-1]	Moment of Inertia [Kg*m^2]
0.1	1.0	2.482	2.532	0.296
0.1	1.0	2.488	2.525	0.297
0.1	1.0	2.512	2.501	0.303
0.1	0.8	2.832	2.219	0.309
0.1	0.8	2.818	2.23	0.305
0.1	0.8	2.788	2.254	0.299
0.1	0.6	3.458	1.817	0.346
0.1	0.6	3.343	1.879	0.323
0.1	0.6	3.534	1.778	0.362

Figure 2: The results from the first experiment with constant mass. Columns *Angular Velocity* and *Moment of inertia* are calculated values.

Mass [kg]	Height [m]	Average Period [s]	Angular Velocity [s^-1]	Moment of Inertia [Kg*m^2]
0.1	1.0	2.482	2.532	0.296
0.1	1.0	2.488	2.525	0.297
0.1	1.0	2.512	2.501	0.303
0.2	1.0	1.799	3.493	0.301
0.2	1.0	1.761	3.568	0.288
0.2	1.0	1.785	3.52	0.296
0.5	1.0	1.151	5.459	0.278
0.5	1.0	1.14	5.512	0.272
0.5	1.0	1.147	5.478	0.276

Figure 3: The results from the first experiment with constant height. Columns *Angular Velocity* and *Moment of inertia* are calculated values

Mass [kg]	Height [m]	Time [s]	Moment of Inertia [Kg*m^2]
0.1	1.0	2.35	0.267
0.1	1.0	2.31	0.258
0.1	1.0	2.37	0.272
0.1	0.8	2.18	0.288
0.1	0.8	2.03	0.248
0.1	0.8	2.06	0.256
0.1	0.6	1.71	0.235
0.1	0.6	1.73	0.24
0.1	0.6	1.81	0.264

Figure 4: The results from the second experiment with constant mass. Column *Moment of inertia* is calculated.

Mass [kg]	Height [m]	Time [s]	Moment of Inertia [Kg*m^2]
0.1	1.0	2.35	0.267
0.1	1.0	2.31	0.258
0.1	1.0	2.37	0.272
0.2	1.0	1.66	0.256
0.2	1.0	1.81	0.309
0.2	1.0	1.71	0.273
0.5	1.0	1.13	0.269
0.5	1.0	1.13	0.269
0.5	1.0	1.11	0.258

Figure 5: The results from the second experiment with constant height. Column *Moment of inertia* is calculated.

3.2 Second Experiment

For the second experiment, we went through the same procedure as experiment 1. We conducted the experiment 5 times, to compare what happens when the mass stays constant, and when the height stays constant, as can be seen in Figure 4 and Figure 5. Here the time measurement is not as accurate as in the first experiment, since we could not compensate for our error by taking the average of the time it takes for the mass to fall. We once again measured the height by ruler, so the error there should be about the same.

3.3 Third Experiment

We conducted the third experiment with 3 different masses. Here we took the average of the gyroscope's period over 30 rotations, and the average of the precession period over 4 rotations.We did this in the aim of minimizing the error in our measurements. We also conducted each trial 5 times, in order to get more accurate results, as can be seen in Figure

Mass [kg]	Gyroscope Period [s]	Precession Period [s]	Moment of Inertia [Kg*m^2]
0.2	0.755	19.975	0.277
0.2	0.589	21.97	0.238
0.2	0.587	19.24	0.208
0.2	0.493	26.48	0.24
0.2	0.631	21.325	0.247
0.5	1.036	15.54	0.74
0.5	0.577	8.65	0.229
0.5	0.577	8.66	0.23
0.5	0.519	7.93	0.189
0.5	1.135	5.34	0.279
1.0	0.591	4.97	0.27
1.0	0.675	4.43	0.275
1.0	0.529	5.458	0.266
1.0	0.643	4.62	0.273
1.0	0.591	5.268	0.286

Figure 6: The results from the third experiment. Column *Moment of inertia* is calculated.

6. We measured the length between the pivot bearing anf the mass to be $l = 0.37m$.

4 Data Analysis

We calculated the error for the moment of inertia using the Gauss method for error propagation:

$$\triangle f = \sqrt{\sum_i (\frac{\partial f}{\partial x_i})^2 \triangle x_i^2}$$

where f is the function used to calculate the moment of inertia and x_i are the different variables f takes. When possible, we calculated the error for our measurements by calculating the standard derivation for the trials of each experiment. In our case, we used this method for the average period time in experiment 1 and the time it took for the mass to fall in experiment 2. For the error in measuring the height in experiments 1 and 2, measuring the length from the mass to the pivot joint in experiment 3, as well as the periods in experiment 3, we had to come up with a realistic error for each measurement. We set the absolute error of the height in experiments 1 and 2, and the absolute error of the length between mass and pivot bearing in experiment 3 to be $\triangle h = \triangle l = 0.002m$ We set the absolute error for the gyroscope's period and the precession period in experiment 3 to $\triangle T_\omega = \triangle T_\Omega = 0.04s$. For the other variables, we set the absolute error to $\frac{1}{2}$ of the last significant digit.

As can be seen in Figure 7, the results for the average moment of inertia from each experiment correspond to our expectations and all lie within the realm of uncertainty.

	Average Moment of Inertia [Kg*m^2]	Error	Error Percentage
Experiment 1	0.303	0.025	8.214
Experiment 2	0.264	0.039	14.934
Experiment 3	0.283	0.018	6.387

Figure 7: Average moment of inertia from each experiment

5 Discussion

Our results confirm the law of conservation of energy and the equation used to calculate the angular momentum of the precession of a gyroscope. We also observed that certain ways to measure the moment of inertia are more accurate than others. In particular the methods in experiment 2 lead to a bigger uncertainty as the other two: An error range of 15% in experiment 2 as compared to 8% and 6% in experiments 1 and 3 respectively. We suspect this to be due to the bigger error in the time measurements in experiment 2, as we couldn't average out the time for the mass to fall for each run, compared to being able to measure the time for multiple rotations of the wheel to get an average in experiments 1 and 3.

6 Conclusion

In conclusion, the results from each experiment coincide which each other, showing that the moment of inertia of a rapidly rotating symmetric gyroscope can be calculated a number of ways. One way we could improve the results of the experiment would be by using more accurate tools, for example using lasers to measure the time it took for the mass to fall as well as giving us a more accurate height measurement. We could further our research into angular momentum and the physics behind a gyroscope by observing non symmetric rotating gyroscopes.