17 - Geometrical Optics

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Abstract

We measured the focal length of two converging lenses and a diverging lens. Moreover, we determined the grating constant of two net wires of coarse and fine grating, respectively. Lastly, we verified Abbe's imaging theory. Our results are quite accurate, however they do not lie within the range of their calculated errors. This has to do with the fact that we had to estimate, for one, the point at which the images were focused and, for another, the slit width, at which the vertical lines of the grating vanished. By improving the determination of these points better values can be gained.

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1 Introduction

Geometrical optics is based on the assumption of light rays that propagate linearly in homogeneous and isotropic media and follow the law of reflection as well as Snell's law of refraction. For optical imaging, every light beam in the first medium has to be correlated with a light beam in the final medium. In addition, all the rays emanating from one point might have to re-unite at one point. For spherical lenses, this is the case for small opening angles of the beam and monochromatic light. In addition, the merging light beams cannot cancel each other by interference and the different light rays all have to be in phase at the image point. This condition is automatically satisfied by Fermat's principle, which states that the different light paths of an image have to be at equal length for them to simultaneously be an extremum. As shown in figure 1, the image created by a spherical lens will be refracted twice δ is the outgoing beam of a thin lens and is given as $\delta = \alpha_1 + \alpha_2 = (n-1)(\beta_1 + \beta_2)$, where $\alpha_1, \alpha_2, \beta_1$ and β_2 are the angles indicated in the figure.

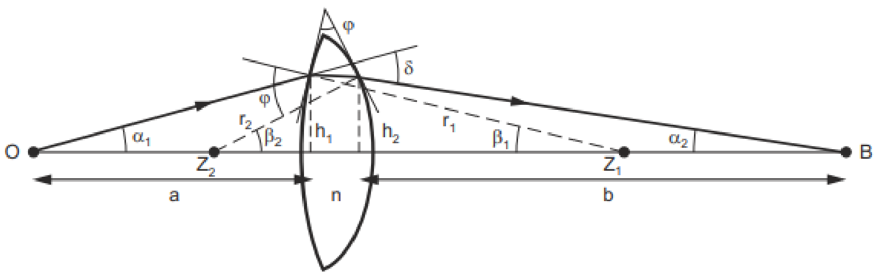

Figure 1: Setup for the lens equation. O is the object, B is the image of the object, $\alpha_1, \alpha_2, \beta_1$ and β_2 are angles, n is the refractive index of the lens, r_1 and r_2 are the curvature radii, a is the distance between the object and the lens, and b is the image distance (source: [1])

Considering a thin lens and an infinite distance to point O, we get the lens equation

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f}. \tag{1}$$

The magnification scale v is further given through

$$v = \frac{b}{a}. \tag{2}$$

If the image appears in image space, it is called a real image and if it arises in object space, it is called a virtual image. We can create a centered lens system by setting multiple lenses behind each other such that their optical axes coincide. In this case, the focal length of the system is given as

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}, \tag{3}$$

where f_1 and f_2 are the focal lengths of the individual lenses.

Bessel had another method of determining the focal length of a lens. For d the fixed distance between object and image, the lens will produce a magnified image from position 1 and a demagnified image from position 2. Using the displacement e of the lens, we get the focal length through

$$f = \frac{d^2 - e^2}{4d}. \tag{4}$$

For diverging lenses with $f < 0$, the focal length can be determined by adding a converging length of known focal length and creating a centered lens system.

Abbe's imaging theory looks at the influence of diffraction in optical imaging. Consider figure 2. An object G (such as a wire netting) is imaged to a screen S through a lens L. When inserting an aperture B, the image will be modified, contrary to what one would deduce from geometrical optics. This has to do with the grating caused by the wire netting. At certain angles α_p the intensity maxima can also be observed behind the grating. Each α_p is related to a bright spot in the focal plane of L. The distance d between two adjacent points can thus be determined through

$$d \approx f \lambda g, \tag{5}$$

where g ($[g] = \frac{1}{\text{cm}}$) is the grating constant and λ the wavelength of the light beam. If the aperture introduced is smaller than d, the image disappears.

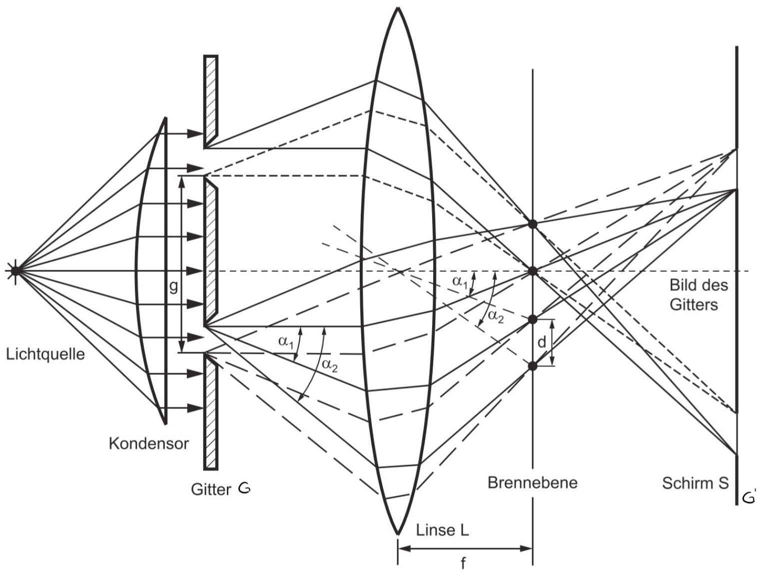

Figure 2: Setup for the verification of Abbe's theorem (source: [1])

2 Experiment

The experiment consisted of four parts. We used a LED light source of wavelength 525nm, a ruler, two converging lenses, a diverging lens, and two wire nets - one with a coarse grating and one with a fine grating.

Part 1: Focal length of converging lenses We measured the focal length of two converging lenses in two different ways. For the first method we set up an object, the lens and a screen, such that the lens focuses the image of the object onto the screen. We then considered the object distance b and image distance a and calculated the lenses' focal length f using 1. As a second method, we used Bessel, i.e. equation 4.

Part 2: Focal length of diverging lenses We determined the focal length of a diverging lens. For this, we stuck the diverging lens together with the two converging lenses whose focal length we measured previously and did the same procedure as in part 1. Using equation 3 we then calculated the diverging lenses' focal length.

Part 3: Grating constant We imaged two wire nets on a screen using the long-focus converging lens. After measuring the grating constant g' of the image, we calculated the grating constant g via its magnification given by 2.

Part 4: Abbe's imaging theory We verified Abbe's imaging theory. For this, we, again, imaged two wire nets on a screen using the long-focus converging lens. Then, we introduced a vertical slit in the focal plane, for which the width was adjusted until the vertical lines in the figure disappeared. The critical slit width was then measured, by magnifying the slit with the long-focus lens, and measuring its image. We then compared it to the calculated value from equation 5.

3 Results

Part 1: Focal length of converging lenses The measured lengths a and b and the calculated focal length f are shown in table 1. For each lens we measured two setups, their averages $\langle f \rangle$ are also shown. For the Bessel method, the results are shown in table 2.

Lens Nr.	a [cm]	b [cm]	f [cm]	$\langle f \rangle$ [cm]
1	5.85 ± 0.05	64.15 ± 0.05	5.36 ± 0.04	5.32 ± 0.03
	6.85 ± 0.05	23.15 ± 0.05	5.29 ± 0.03	
2	24.50 ± 0.05	35.50 ± 0.05	14.50 ± 0.02	14.70 ± 0.02
	19.80 ± 0.05	60.20 ± 0.05	14.90 ± 0.03	

Table 1: Object distance a, image distance b and focal length f calculated with 1 for two lenses. The measurements were done for two setups, $\langle f \rangle$ is their average.

Lens Nr.	d [cm]	e [cm]	f [cm]
1	40.00 ± 0.05	27.10 ± 0.05	5.41 ± 0.03
2	80.00 ± 0.05	40.70 ± 0.05	14.82 ± 0.02

Table 2: Measurements for the Bessel method. Focal length f is calculated with 4.

Part 2: Focal length of diverging lenses We measured the focal length of a diverging lens twice, once using lens 1 from part 1 and once using lens 2:

$$f_{\text{div},1} = -63.0 \pm 6.0\text{cm}$$

$$f_{\text{div},2} = -58.4 \pm 0.4\text{cm}.$$

Part 3: Grating constant In both the measurements of the grating constants of the wire nets 1 and 2, we used the long-focus lens 2. For wire net 1, we measured the image grating to be $g'_1 = 2.00 \pm 0.02\frac{1}{\text{cm}}$. The second wire net has an image grating of $g'_2 = 4.8 \pm 1.1\frac{1}{\text{cm}}$. Using the magnification given by 2 we obtained the grating constants:

$$g_1 = 6.53 \pm 0.01\frac{1}{\text{cm}}$$

$$g_2 = 15.66 \pm 0.03\frac{1}{\text{cm}}.$$

Part 4: Abbe's imaging theory We measured the widths of the critical slit widths of the two gratings to be $d'_1 = 0.18 \pm 0.05\text{cm}$ and $d'_2 = 0.20 \pm 0.05\text{cm}$. By calculating the magnification with 2 we the calculated the real width of the slits $d_{\text{meas.}}$ and using 5 we calculated a theoretical value. The values can be seen in table 3.

Net no.	$d_{\text{exp.}}$ [μm]	$d_{\text{theor.}}$ [μm]
1	10 ± 30	50 ± 5
2	110 ± 30	120 ± 30

Table 3: Experimental and theoretical critical slit width of wire nets 1 and 2.

4 Data analysis

The error in the experiment comes from the uncertainty of the length measurements. Given that the ruler had a precision of 1mm, we took an uncertainty of 0.5mm on every reading of the image and object distances. Propagation errors were calculated using Gaussian error propagation. Another error appeared in part four of the experiment. While adjusting the slit width to make the vertical lines disappear, it was rather difficult to determine the exact point at which the vertical lines could no longer be seen. Hence, the slit width might be slightly wider or narrower than it should be.

5 Discussion

Overall all measurements and calculations yielded fairly accurate and correct results. In part 1 both methods resulted in very similar values for the focal lengths of the converging lenses, well within the margin of error given by focusing the images by eye.

In the second part, when measuring the focal length of the diverging lens, using the long-focus length lens resulted in a much smaller error, as object and screen were way further apart, which allowed for a much more precise placement of the lens.

The grating constants g_1 and g_2 determined in part 3 gave us theoretical values for the critical slit width $d_{\text{theor.}}$, which were in the same order of magnitude as determined experimentally in part 4, but they were still about 100 to 200% bigger. Most of the error most certainly lies in the experimentally determined values $d_{\text{exp.}}$. It was very hard to see exactly when the vertical lines in the image disappeared, which lead to a big error.

6 Conclusion

We successfully determined the focal length of two converging lenses and a diverging lens, although the results do not exactly within their calculated errors, which is due to the fact that we had to focus the image by eye. For the verification of Abbe's theorem, we have values of the correct magnitudes, but the errors are too big since determining the point at which the vertical lines disappear was rather difficult. The results could be improved by, for example, auto-focusing the images instead of doing it by eye. Further, making it easier to see when the vertical lines disappear in the verification of Abbe's theory, the corresponding values could be better.

References

[1] https://ap.phys.ethz.ch/Anleitungen/Bilingual/17_Manual.pdf

7 Appendix

ETHZ **Physikalisches Praktikum**

Name 1: Manuel Antoinette **Datum:** 21.03.22

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17	Geometrische Optik und ihre Grenzen

1. Brennweite zweier Sammellinsen

alle Längenangaben in der Einheit cm

a) aus Objekt- und Bilddistanz
(jeweils zwei unterschiedliche Objektdistanzen wählen)

Linse Nr.	a	b	f	⟨f⟩
1	5.85	64.15	5.36 ± 0.04	5.324 ± 0.026
	6.85	23.15	5.286 ± 0.060	
2	24.5	35.5	14.496 ± 0.19	14.698 ± 0.017
	19.8	60.2	14.900 ± 0.028	

b) nach Bessel

Linse Nr.	d	e	f
1	40.0	27.1	5.410 ± 0.025
2	80.0	40.7	14.823 ± 0.020

2. Brennweite einer Zerstreuungslinse

	a	b	f_1	f	f_2
L_1	6.4	63.6	5.324 ± 0.026	5.81 ± 0.04	-63.0 ± 6.0
L_2	25.6	84.4	14.698 ± 0.017	19.642 ± 0.030	-58.4 ± 0.4

3. Gitterkonstante des Drahtnetzes

Netz-Nr.	Linse Nr.	a	b	G' $\frac{1}{cm}$	G $\frac{1}{cm}$
1	2	25.8	84.2	2.00 ± 0.2 (0.5)	6.527 ± 0.013
2	2	25.65	84.35	4.8 ± 1.1 (0.21)	15.660 ± 0.032

4. ABBEsche Abbildungstheorie

λ = 525 nm Linse Nr. = 2 Abstand Spalt - Linse: 14.7 cm

Kritische Spaltbreite durch Abbildung und Rechnung:

Netz-Nr.	a cm	b cm	d´ cm	$d_{exp.}$ m	$d_{berechnet}$ m
1	15.2	279.8	0.18	$(9.8 \pm 2.7) \cdot 10^{-5}$	$(5.0 \pm 0.5) \cdot 10^{-5}$
2	15.3	279.7	0.2	0.000109 ± 0.000027	0.000121 ± 0.000029

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