Entangled Photons

Manuel Antoinette

March 21, 2023

Using barium borate crystals and commercial single-photon counting modules, we build a setup that allows us to determine the S measure introduced by Bell and CHSH [1] [2]. The BBO crystal downconverts a vertically polarized photon into two horizontally polarized ones, and vice versa. Firstly, we perform a time correlation measurement, to determine the accidental coincidence counts, which we found to be 364 ± 20 per 10 s. Secondly, we measure the momentum correlation to determine the optimal angle to measure the downconverted photons from. We determined an angle of $2.86°$ to be optimal. Finally, to calculate the S value, we first diagonally polarize the laser beam to generate photons in a Bell state. Then, we measure the photon counts for various polarizer angles in front of the SPCMs. We determined a value of $S = 1.897 \pm 0.028$, which unfortunately is smaller than 2, therefore doesn't violate local realism. We argue that the reason for this lies mainly in an imperfect Bell state, as it was hard to determine the exact state of the light beam with tool provided in our setup.

Contents

Laser Safety Notice

In this lab we work with an 405 nm laser beam with a maximum power of 20 mW. Even the 5 mW of power used in the experiments can severely damage the eyes. Appropriate laser safety measurements with UV-blocking goggles are required.

1 Introduction

When quantum mechanics was first introduced it gained much attention and praise, as it was able to accurately describe previously poorly understood phenomena, such as the spectral lines of Helium. It then became clear though, that at the same time we had to let go of some principles of our understanding of nature. The basis of many non-classical phenomena is the entanglement of particles. The first hypothesis was set up by Bell in 1964 [1] and yielded small violations. These tests were later improved upon by Clauser, Horne, Shimony and Holt [2]. Nowadays, we view entanglement as a fundamental concept that powers modern applications such as quantum computing.

In this Lab we aim to perform such an abovementioned test. Using barium borate crystals we create two entangled photons and then measure their polarization correlations. By setting the photons up in a Bell state, we can show that the CHSH measure S to be greater than two, which is not possible with any theory consistent with local realism.

2 Theory

2.1 Polarization of light

The polarization of a single photon has one degree of freedom between two states. Therefore, any polarization state $|\psi\rangle$ can be represented as a complex linear combination of the two orthonormal states $|H\rangle$ and $|V\rangle$, denoting the horizontal and vertical polarization:

$$|\psi\rangle = a\,|V\rangle + b\,|H\rangle\,,$$

where $a, b \in \mathbb{C}$ and $|a|^2 + |b|^2 = 1$. We can also represent $|\psi\rangle$ in polar coordinates:

$$|\psi\rangle = \cos(\theta)\,|V\rangle + e^{i\varphi}\sin(\theta)\,|H\rangle\,,$$

with $\theta \in \mathbb{R}$ giving the polarization direction and $\varphi \in \mathbb{R}$ representing the phase difference between $|V\rangle$ and $|H\rangle$ (global phase is ignored).

A photon encountering a polarizer P_α oriented at an angle α has the following probability of passing it:

$$|\langle P_\alpha|\psi\rangle|^2 = \cos(\theta-\alpha)^2.$$

The state of the polarization of two photons is again given by the linear combination of all combinations of $|V\rangle$ and $|H\rangle$ of each photon:

$$\begin{aligned}|\psi_{AB}\rangle = \quad & \gamma\,|V\rangle_A\,|V\rangle_B\\ & +\eta\,|V\rangle_A\,|H\rangle_B\\ & +\varepsilon\,|H\rangle_A\,|V\rangle_B\\ & +\zeta\,|H\rangle_A\,|H\rangle_B\,,\end{aligned}$$

with $\gamma, \eta, \varepsilon, \zeta \in \mathbb{C}$. If a state can be written as a tensor product of two single-photon states $|\psi_{AB}\rangle = |\psi\rangle_A\,|\psi\rangle_B$, it is called factorizable and possesses no entanglement. For entangled two-particle states, if we know the state of one particle, we also have non-trivial information about the other photon. The so-called Bell state is maximally entangled:

$$\begin{aligned}|\psi_{\text{Bell}}\rangle &= \sqrt{\frac{1}{2}}\left(|V\rangle_A\,|V\rangle_B + |H\rangle_A\,|H\rangle_B\right)\\ &= \sqrt{\frac{1}{2}}\left(|VV\rangle + |HH\rangle\right).\end{aligned}$$

2.2 Downconversion

In order to create two photons entangled in a Bell state, we use spontaneous parametric downconversion (SPDC). This occurs in crystals with non-linear susceptibility. The microscopic description is quite complex, but for our purposes it suffices to know that these crystals convert an incoming photon into two entangled photons with twice the wavelength. The polarization of the entangled photons depends on the downconversion type:

- Type-I downconversion: Two photons of equal polarization are generated: $e \to oo$ or $o \to ee$.
- Type-II downconversion: Two photons of equal polarization are generated: $e \to eo$ or $o \to eo$.

e and o signify whether the polarization is aligned (ordinary) or misaligned (extraordinary) with the crystallographic axis. Further, these crystals also have dispersion and relatively strong birefringence effects, affecting the phase shift of the photons.

Here, we will use a barium borate (BBO) crystal, which performs type-I downconversion. To create a Bell state of two entangled photons, we therefore pump two BBO crystals with their crystallographic axes aligned horizontally and vertically with diagonally polarized light:

$$\sqrt{\frac{1}{2}}\left(|V\rangle + |H\rangle\right) \;\to\; \sqrt{\frac{1}{2}}\left(|HH\rangle + |VV\rangle\right)$$

As one of the BBO crystal is behind the other one, the birefringence effect onto the $|HH\rangle$ component is not equal to $|VV\rangle$, we therefore get a phase shift δ in the entangled state after the two crystals:

$$\cos(\theta)\,|HH\rangle + e^{i(\varphi+\delta)}\sin(\theta)\,|VV\rangle$$

We can compensate for δ by inducing a phase shift $\varphi = -\delta$ into the pump beam.

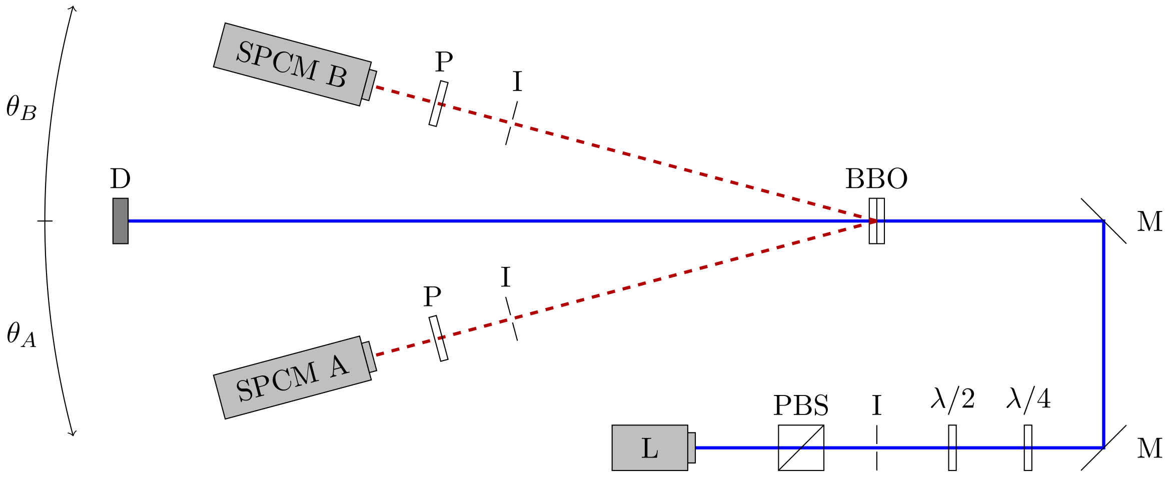


Figure 1: Schematic of the laser setup: 405 nm laser (L), polarizing beam splitter (PBS), iris (I), $\lambda/2$ and $\lambda/4$ waveplates, Mirrors (M), barium borate crystals (BBO), beam dump (D), polarizers (P), single-photon counting modules (SPCM A) and (SPCM B) oriented at angles θ_A and θ_B.

2.3 Bell's inequality

To test the violation of locality and realism by quantum mechanics, Bell came up with a measure S that was later refined by Clauser, Horne, Shimony and Holt (CHSH) [2].

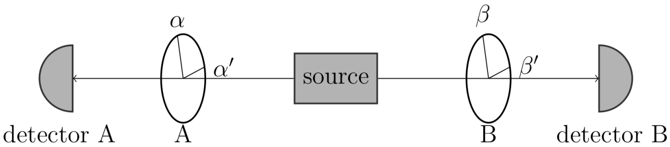


Figure 2: Correlation experiment

Consider the experimental setup shown in figure (2). The source simultaneously emits two photons in each direction. The polarization of the photons is then measured with two polarizers oriented at an angle α and β on either side. The two detectors A and B then measure the number of photons that make it past the polarizers.

The measure S is then given by

$$S = E(\alpha, \beta) - E(\alpha', \beta) + E(\alpha, \beta') + E(\alpha', \beta'),$$

where $E(\alpha^{(\prime)}, \beta^{(\prime)})$ are correlation coefficients:

$$\begin{aligned} &E(\alpha, \beta) \\ &\quad = P(\alpha, \beta) + P(\alpha^\perp, \beta^\perp) - P(\alpha^\perp, \beta) - P(\alpha, \beta^\perp) \\ &\quad = \frac{N(\alpha, \beta) + N(\alpha^\perp, \beta^\perp) - N(\alpha^\perp, \beta) - N(\alpha, \beta^\perp)}{N(\alpha, \beta) + N(\alpha^\perp, \beta^\perp) + N(\alpha^\perp, \beta) + N(\alpha, \beta^\perp)} \end{aligned}$$

$P(\alpha, \beta)$ is the probability of simultaneously detecting a photon at detector A and B with the polarizers angled at α and β, respectively. $N(\alpha, \beta)$ is the number of simultaneous detections at angles α and β.

CHSH showed that for any theory consistent with local realism

$$|S| \leq 2$$

hold, but for quantum mechanics

$$|S| \leq 2\sqrt{2}.$$

For two photons in a Bell state, S is maximal for a certain set of $\alpha^{(\prime)}$ and $\beta^{(\prime)}$.

3 Setup

A schematic of the laser setup is shown in figure (1). We use a 405 nm AlGaN semiconductor diode laser (L) with a maximal power output of 20 mW at a current of 40 mA. In the following experiments we will be operating the laser at around 5 mW at 10 mA. The beam has a diameter of about 2 mm and a spectral width of 0.5 nm. Directly after the laser a polarizing beam splitter (PBS) horizontally polarizes the beam, the reflected vertically polarized beam is blocked. After an iris (I) $\lambda/2$ and $\lambda/4$ waveplates are used to set the polarization direction and phase shift. The beam is then reflected off two mirrors in order to give us precise control over the beam direction through the BBO crystal and into the two single-photon counting modules (SPCM A) and (SPCM B) angled at θ_A and θ_B. In front of each SPCM there is an iris and a polarizer, which is oriented at α and β respectively, analogous to the setup in figure (2). Finally, the beam is sent into a beam dump (D), which is stack of tightly packed razor blades, that blocks nearly all back reflection.

3.1 BBO crystal

The BBO crystal downconverts the 405 nm photons into pairs of 810 nm photons. It is made from an assembly of two β-BaB_2O_4 crystals measuring 5 mm × 5 mm × 0.5 mm and are cut at 28.7° with respect to their crystallographic c-axis, such that we get type-I phase matching for the two downconverted photons.

3.2 SPCMs

To detect single photons, a commercial single photon counting module built around an avalanche photodiode operated in Geiger mode is used. They have a dark count rate of about 700 counts per second, an active area of 180 µm diameter and a quantum efficiency of 60 %. A detected photon is signalled by an electrical pulse of around 16 ns duration, followed by a 35 ns dead time, where no photons can be detected. This leads to saturation of about 25×10^6 counts per second. To optimize the counting, each SPCM is mounted behind a light-tight cage containing a long pass filter and a lens assembly focussing the beam of downconverted photons onto the active area of the SPCMs. The filter transmits 0.001 % at 640 nm and below, 50 % at 780 nm, 80 % at 810 nm and 90 % at 850 nm. The lenses have a focal length of 75 mm and can be finely adjusted with a translation holder.

To count the cases where two photons (a downconverted pair from the BBO crystal) hit the two detectors simultaneously, a coincidence detection circuit is used, consisting of two D-type flip-flops. Roughly, it works by delaying the signal from one of the SPCMs by around 6 ns, and then counting the number of rising edges of the delayed signal, while the other signal is high. After an integration time of a few seconds, the count is set to zero again. Here, we used an integration time of 10 s for all measurements.

4 Experiments

To maximize the coincidence rate with roughly equal single count rates the setup was first carefully aligned using the two mirrors, the orientation of the BBO crystal and the lens array in front of each SPCM. Once aligned, we closed the irises in front of the SPCMs to a diameter of around 5 mm, and the iris right after the PBS, as far as the photon counts were not affected, around 2 mm. The photon counts without any polarizers in front of the SPCMS can be seen table (1).

	counts [per 10 s]
SPCM A	447400 ± 6900
SPCM B	454800 ± 2600
Coincidences	7512 ± 48

Table 1: Photon count number after aligning the setup, without polarizers.

4.1 Time correlation

In order to measure the time correlation of the photon pairs, we varied the length of the coaxial cable connecting SPCM A to the coincidence detection circuit, and therefore inducing a timeshift between the signals from SPCM A and SPCM B. The speed of light in the RG-58U cables used, is 2/3 of that in vacuum. The (additional) cable lengths and their corresponding delays are shown in table (2).

additional cable length [m]	delay [ns]
0	0
0.28	1.4
0.53	2.65
0.81	4.05
1.04	5.20
1.32	6.60
1.57	7.86
1.85	9.26

Table 2: Cable lengths and corresponding delays used in time correlation measurement.

4.2 Momentum correlation

Next, we measured the photon counts for various angles θ_A, θ_B. For each we chose three angles, resulting in a total of 9 combinations:

$$\theta_A, \theta_B \in \{1.43°, 2.86°, 4.29°\}$$

When changing the angle an SPCM, its lens array had to be refocused, in order to maximize its photon count again. All other alignment parameters (Mirrors, BBO crystal) were held constant though.

4.3 Test of local realism

Finally, we measured the measure S of a Bell state. To get the downconverted photons to be in a Bell state, we use the $\lambda/2$ waveplate oriented at 22.5° to diagonally polarize the light. Then, we also have to compensate for the phase shift δ induced by the BBO crystal. This phase shift is close to a quarter wave length, which is why we use a $\lambda/4$ waveplate with its fast axis oriented at 45° relative to the incoming diagonally polarized light. To precisely match the phase shift to the BBO crystal, we also slightly spun the $\lambda/4$ waveplate around its vertical axis. This increases the thickness of the waveplate along the beam path, therefore increasing the phase shift. By setting the polarizers in front of the SPCMs to various angles, we could ensure and optimize the Bell state.

With the Bell state established, we started taking measurements for various angles $\alpha^{(\prime)}$ and $\beta^{(\prime)}$ of the polarizers in front of the SPCMs. Our chosen angles were:

$$\begin{aligned} \alpha &= -45°, & \alpha^{\perp} &= 45° \\ \alpha' &= 0°, & \alpha'^{\perp} &= 90° \\ \beta &= -22.5°, & \beta^{\perp} &= 67.5° \\ \beta' &= 22.5°, & \beta'^{\perp} &= 112.5° \end{aligned}$$

For each α-β angle combination we recorded five counts of SPCM A and B and their coincidences with an integration time of 10 s. We also measured the accidental coincidences of the whole setup (with the polarizers) by again delaying the SPCM A signal by 9.26 ns.

5 Results

5.1 Time correlation

The photon counts of SPCM A and SPCM B and their coincidences for various delay times of the SPCM A signal to the coincidence detection circuit can be seen in figure (3). We can see that the coincidence count falls off drastically at around 6 ns, which coincides nicely with the 6 ns window, which the coincidence detection circuit works with. By looking at the coincidence count for delay times after the fall off, we get an estimate for accidental coincidences, resulting not from downconverted photons, but rather two independent photons hitting the SPCMs at the same time. Counts for a delay of 9.26 ns can be seen in table (3).

9.26 ns delay	counts [per 10 s]
SPCM A	437300 ± 6200
SPCM B	451000 ± 7200
Coincidences	346 ± 20

Table 3: Photon count number with a 9.26 ns delay of the SPCM A signal to the coincidence detection circuit.

Figure (3a) also shows that the single photon counts of SPCM A and B stay constant, which is as expected, as the delay time cannot affect the single counts.

5.2 Momentum correlations

The results of the momentum correlation measurements can be seen in figure (4). For the single photon counts, we can clearly see that they were maximal at an angle of around 2.86°. Setting both SPCMs to this angle then also maximizes the coincidence rate, as expected.

5.3 Test of local realism

The counts recorded for the entangled photons can be seen in table (4). The accidental coincidences were around 180 for any α and β. With these numbers we can calculate the measure S:

$$N(\alpha, \beta) = \text{coincidences} - \text{accidentals}$$

$$S = 1.897 \pm 0.028$$

6 Discussion

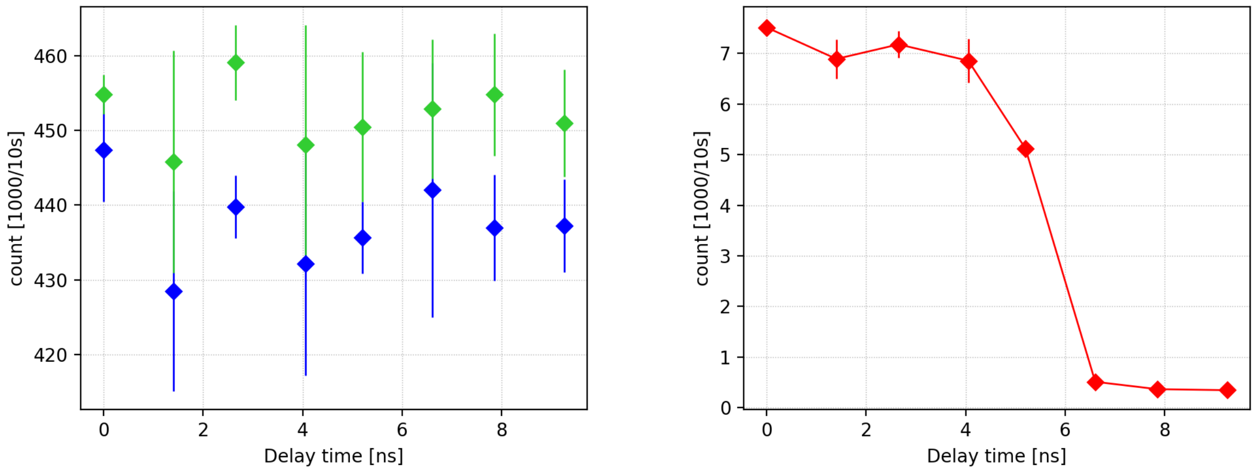
Unfortunately, we could not show the violation of Bell's inequality $S < 2$. Reason for this is most likely an improper Bell state. Using only the SPCM counts and the polarizers it was rather difficult to determine the exact polarization state of the downconverted photons. The photon counts varied greatly, even for long integration times (10 s). This meant that the effects of small adjustments on the setup could only be seen after taking a lot of data, which we didn't have the time to do for all parameters.

The most important parameters for the Bell state are the rotation of the $\lambda/2$ waveplate and the twist of the $\lambda/4$ waveplate. For these we systematically recorded the coincidence rate for various polarizer angles (in front of the SPCMs). The results of which can be seen in figure (5). The goal was to equalize the counts for all polarizer rotations. For the $\lambda/2$ waveplate we settled on a value between 264° and 263°. The twist $\lambda/4$ waveplate was a little harder to adjust, as there was no scale for it. The 3$^{\text{rd}}$ adjustment run was our best attempt at equalizing all counts. It is safe to say though, that this probably isn't a perfectly clean Bell state.

Additionally, errors could have come from an underestimation of the accidental coincidences. Using the single photon counts N_A and N_B we can give a theoretical prediction of the accidentals [3]:

$$N_{\text{Ac.}} = \frac{\tau N_A N_B}{T}, \tag{1}$$

where $T = 10\,\text{s}$ is the integration time and $\tau = 16\,\text{ns}$ is the coincidence window. This gives us a theoretical average accidental rate of 207, which is about 15 %



(a) SPCM A (blue) and SPCM B (green) (b) coincidences (red)

Figure 3: Photon counts for various delay times of the SPCM A signal to the coincidence detection circuit.

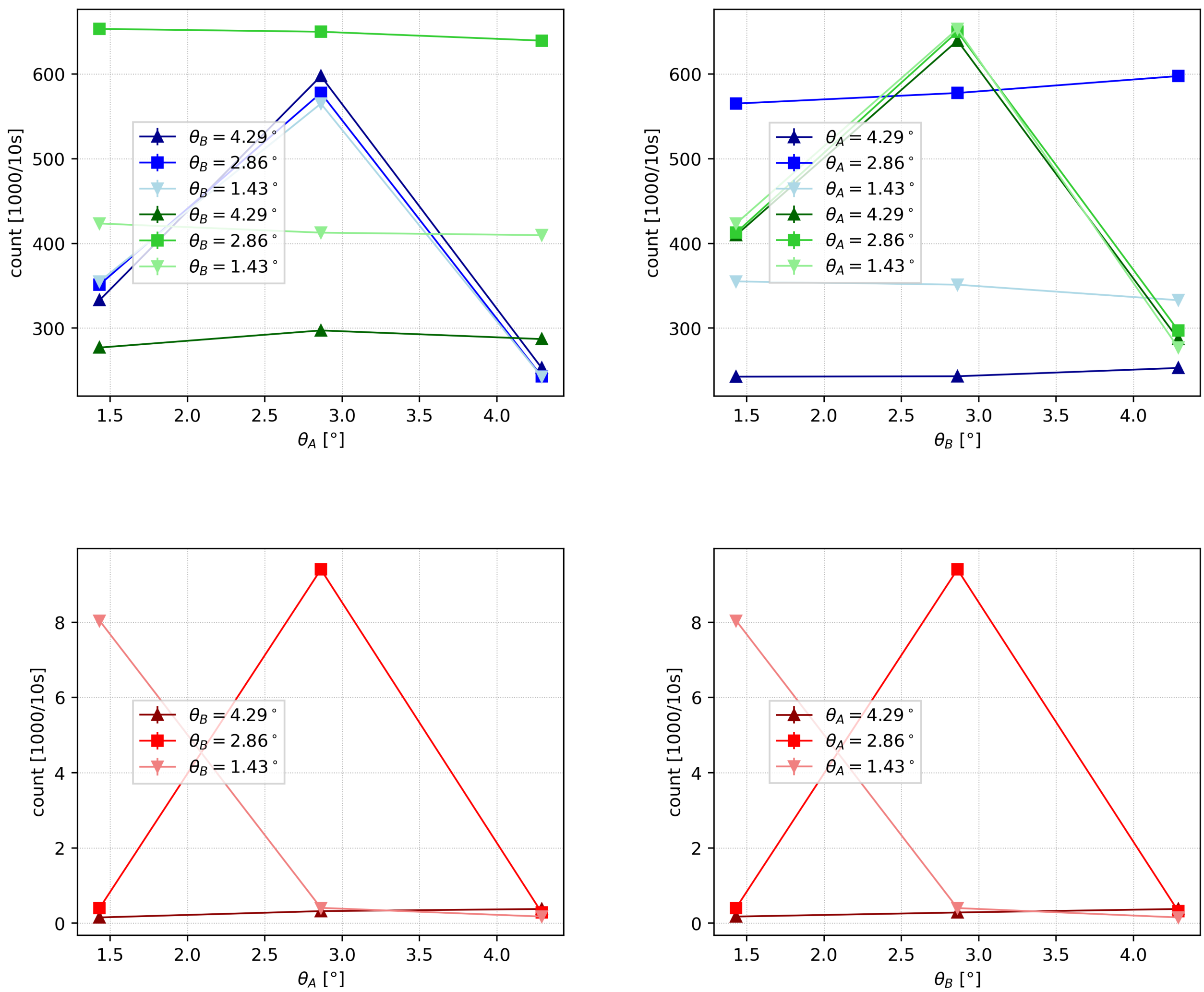


Figure 4: Photon counts of SPCM A (blue), SPCM B (green) and coincidences (red) for various angles θ_A and θ_B.

α	β	SPCM A	SPCM B	coincidences	theoretical accidental coincidences
$-45°$	$-22.5°$	355100 ± 8100	434800 ± 7300	1964 ± 59	247 ± 7
$-45°$	$22.5°$	350700 ± 3200	436700 ± 3200	894 ± 60	245 ± 3
$-45°$	$67.5°$	344300 ± 1200	429200 ± 2300	1053 ± 33	236 ± 2
$-45°$	$112.5°$	340700 ± 1500	427300 ± 1000	2132 ± 40	233 ± 1
$0°$	$-22.5°$	360700 ± 1500	421900 ± 1500	2233 ± 36	243 ± 1
$0°$	$22.5°$	352500 ± 700	415000 ± 1300	2129 ± 19	234 ± 1
$0°$	$67.5°$	347400 ± 1700	408900 ± 1100	783 ± 20	227 ± 1
$0°$	$112.5°$	342400 ± 900	403600 ± 1100	822 ± 14	221 ± 1
$45°$	$-22.5°$	313300 ± 1300	400000 ± 1700	955 ± 27	200 ± 1
$45°$	$22.5°$	302100 ± 1600	391300 ± 4500	2012 ± 20	189 ± 2
$45°$	$67.5°$	299400 ± 700	385900 ± 500	1955 ± 29	185 ± 0
$45°$	$112.5°$	298800 ± 500	387200 ± 400	909 ± 33	185 ± 0
$90°$	$-22.5°$	272500 ± 500	387700 ± 300	717 ± 30	169 ± 0
$90°$	$22.5°$	272700 ± 700	387500 ± 900	779 ± 23	169 ± 1
$90°$	$67.5°$	269500 ± 900	383300 ± 500	2202 ± 40	165 ± 1
$90°$	$112.5°$	268300 ± 500	384100 ± 400	2143 ± 25	164 ± 0

Table 4: Photon counts and coincidences in a 10 s interval of two entangled photons in a Bell state measured each with a polarizer angled at α and β degrees respectively. Errors are the standard deviation of 5 measurements. Theoretical accidental coincidences are calculated using eqn. (1).

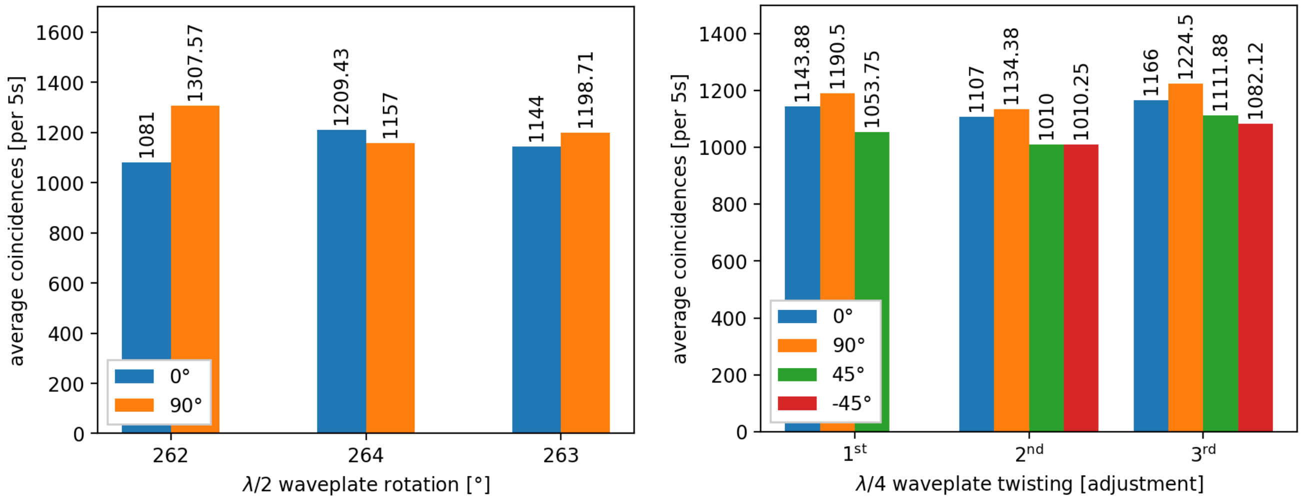


Figure 5: Average coincidences for various polarizer rotations (0°, 90°, 45°, $-45°$) and waveplate adjustments. The goal was to set the waveplates such that the coincidences was equal for all polarization angles.

higher than we measured. Using this accidental rate we get $S = 1.937 \pm 0.029$, which is closer to 2, but still doesn't violate Bell's inequality.

Further, the angles θ_A and θ_B of the SPCMs could also be even more optimized, which could lead to a better result. Finally, a higher beam intensity would also significantly increase the signal-to-noise ratio. This could have been achieved by precessing the polarizing beams splitter with a waveplate to allow more of the original laser beam to pass it.

7 Conclusion

Using barium borate crystals and commercial single-photon counting modules, we built a setup that allowed us to determine the S measure introduced by Bell and CHSH [1] [2]. The BBO crystal downconverts a vertically polarized photon into two horizontally polarized ones, and vice versa. To account for accidental counts from stray photons, we first performed a time measurement and determined there to be around 346 ± 20 accidental coincidence counts per 10 s. Secondly, to find the optimal angle to detect the downconverted photons from the BBO crystal we measured the momentum correlation. The optimal angle was determined to be around 2.86°. Finally, for the S measure, we diagonally polarized the laser beam, which corresponds to a Bell state. Using the photon counts for various angles of the polarizers placed in front of the SPCMs, we determined $S = 1.897 \pm 0.028$, which is smaller than 2, which doesn't violate local realism. We argue that the reason for this lies mainly in an imperfect Bell state, as with our setup it was hard to evaluate the exact state of the laser beam.

References

[1] J. S. Bell (Nov. 1964) *On the Einstein Podolsky Rosen Paradox*, Physics, vol. 1.

[2] J. F. Clauser, M. A. Horne, A. Shimony and R. A. Holt (Oct. 1969) *Proposed Experiment to Test Local Hidden-Variable Theories*, Phys. Rev. Lett. vol. 23.

[3] D. Dehlinger and M. W. Mitchell (2002) *Entangled photons, nonlocality, and Bell inequalities in the undergraduate laboratory*, Am. J. Phys. vol. 70, no. 9.

[4] D. Dehlinger and M. W. Mitchell (2002) *Entangled photon apparatus for the undergraduate laboratory*, Am. J. Phys. vol. 70, no. 9.