9 – Absolute Zero

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Abstract

We use a glass bulb filled with helium gas and measure its pressure at two known temperatures to determine the absolute zero temperature. In the first part of the experiment we calibrate the pressure sensor by linearly interpolating between two known temperatures, ambient and vacuum. Our result for absolute zero is $(-272.9 \pm 0.1)\,°\mathrm{C}$, which is $0.3\,°\mathrm{C}$ off the the real value of $-273.15\,°\mathrm{C}$. Finally we use the setup to measure the temperature of liquid nitrogen and got $(-196.8 \pm 0.1)\,°\mathrm{C}$, which is $1\,°\mathrm{C}$ off the literature value of $-195.8\,°\mathrm{C}$.

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1 Introduction

After Boyle and Mariotte found that the product of pressure p and volume V remains constant when varying p or V of a gas, G. Amontons found that the pressure of gas scales linearly with temperature t: $p(t) = \text{constant} \cdot (t - t_0)$. As the notion of a negative pressure was difficult to imagine, Amontons suspected that an absolute zero temperature exists. A series of experiments yielded that gases can only exist in certain combinations of p, t and particle density n. Thus the relationship

$$p = kn(t - t_0)$$

was concluded, where $k = k_B = 1.381 \times 10^{-23}\,\mathrm{J\,K^{-1}}$ and the absolute zero temperature $t_0 = -273.16\,°\mathrm{C}$ are universal constants. In 1954 the Kelvin scale was introduced, which is analogously defined to the Celsius scale but shifted such that $0\,\mathrm{K} = t_0$.

Firstly we calibrate the pressure sensor by measuring the sensor signal at air pressure (U_L, p_L) and at near vacuum (U_t, p_t). Through linear interpolation the pressure p can be calculated with

$$p = p_0 + CU, \tag{1}$$

where

$$C = \frac{p_L - p_t}{U_L - U_t}, \tag{2}$$

and

$$p_0 = p_t - CU_t. \tag{3}$$

Secondly we determine the absolute zero temperature by measuring the pressure of helium gas at two different, known temperatures (melting and boiling point of water) at a fixed volume. This gives us two pressure-temperature measurements pairs (t_k, p_k) and (t_E, p_e) for the boiling and melting point of water respectively. This allows us to obtain an approximate value for the absolute zero temperature:

$$t_{00} \approx -\frac{p_E}{p_k - p_E} t_k. \tag{4}$$

A more accurate value can be obtained, when considering the parasitic volume of the setup, given as a fraction of the whole volume $\frac{V_s}{V} = \varepsilon = 0.001$, and the thermal expansion of the glass tube, calculated via its cubic thermal expansion coefficient $\gamma = 1 \times 10^{-5}\,°\mathrm{C}^{-1}$. The more accurate absolute zero temperature t_0 can then be calculated as follows:

$$a = (1+\varepsilon)p_E - (1+\varepsilon+\gamma t_K)p_K \tag{5}$$
$$b = \varepsilon(p_K - p_E)t_K + (1+\gamma t_K)p_K t_L - p_E(t_L + t_K) \tag{6}$$
$$c = p_E t_L t_K \tag{7}$$
$$t_0 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \tag{8}$$

Finally, to measure the temperature of liquid nitrogen, we can use the setup as a thermometer. For this we fully submerge the bulb in liquid nitrogen and let the gas inside reach thermal equilibrium. With (1), we can calculate the pressure and then with

$$t_{N0} \approx \left(1 - \frac{p_N}{p_E}\right) t_0 \tag{9}$$

we get an approximate value for the temperature t_{N0}. We can again consider the parasitic volume and the thermal expansion of the setup and calculate a more accurate value as follows:

$$A = \frac{p_E}{t_E - t_0} + \frac{\varepsilon(p_E - p_N)}{t_L - t_0} \tag{10}$$
$$t_N = \frac{At_0 + p_N}{A - \gamma p_n}. \tag{11}$$

2 Experiments

Calibration of the pressure sensor Firstly, we disconnected all the tubes from the pressure sensor, exposing it to ambient pressure. The ambient pressure p_L was determined using a mercury barometer. We then took note of the voltage output U_L of the sensor, which gives us the first measurement pair (U_L, p_L). Next, we connected a vacuum pump to the bulb and let it run until the sensor output signal stopped decreasing. This took around 10 minutes. We now have a second measurement (U_t, p_t).

Determining absolute zero Before taking measurements, we have to fill the bulb with helium gas. From the previous step the bulb is already evacuated. Therefore we simply filled the bulb by connecting it to a balloon filled with helium, which gives a constant pressure to the system. To make sure there is no remaining air in the bulb, we evacuated it a second time and filled it with helium again.

For the first measurement, we heated up the bulb to the boiling point of water by placing inside of a steam chamber. Once the pressure sensor signal stabilized, we took the measurement (t_k, p_k).

In the second measurement we cooled the bulb down to the freezing point of water by placing it inside an ice bath. Again, once the sensor signal stabilized, we took the measurement (t_E, p_E).

Temperature of liquid nitrogen To determine the temperature of liquid nitrogen, we simply submerged the bulb in liquid nitrogen and measured the pressure.

3 Results

Calibration of the pressure sensor The mercury barometer showed $(718.00 \pm 0.05)\,\mathrm{mmHg}$ at $(22.5 \pm 0.5)\,°\mathrm{C}$. After the correction of $2.99\,\mathrm{mmHg}$ due to the thermal expansion of the mercury, we get a value of $(715.01 \pm 0.05)\,\mathrm{Torr}$ which translates to

$$p_L = (95\,327 \pm 7)\,\mathrm{Pa}$$

for the ambient pressure. At this pressure, the sensor gave an output voltage of

$$U_L = (-102.520 \pm 0.005)\,\mathrm{mV}.$$

For the evacuated bulb, which we assume to be at $p_t = (10 \pm 10)\,\mathrm{Pa}$, the output voltage was

$$U_t = (-32.880 \pm 0.005)\,\mathrm{mV}.$$

Using (2) and (3) we get $C = (703\,900 \pm 100)\,\mathrm{Pa\,V^{-1}}$ and $p_0 = (23\,146 \pm 4)\,\mathrm{Pa}$. We can now determine the pressure from the sensor data using (1).

Determining absolute zero The boiling point of water at the measured room temperature $t_L = (22.5 \pm 0.5)\,°\mathrm{C}$ is $t_k = (98.30 \pm 0.05)\,°\mathrm{C}$, according to a table provided by the lab. At this temperature the sensor read $U_k = (102.370 \pm 0.005)\,\mathrm{V}$, which translates to a pressure of

$$p_k = (95\,210 \pm 10)\,\mathrm{Pa}.$$

For the bulb in the ice bath we assume the temperature to be $t_E = (0.00 \pm 0.01)\,°\mathrm{C}$. The sensor read $U_E = (66.680 \pm 0.005)\,\mathrm{mV}$, which means that the pressure is

$$p_E = (70\,086 \pm 10)\,\mathrm{Pa}.$$

Using (4) we get an approximate value of

$$t_{00} = (-274.2 \pm 0.2)\,°\mathrm{C}$$

for the absolute zero temperature, which is off by about $1\,°\mathrm{C}$ from the real absolute zero value of $-273.15\,°\mathrm{C}$. Considering the parasitic value and thermal expansion, we get a more accurate value of

$$t_0 = (-272.9 \pm 0.2)\,°\mathrm{C},$$

which is now off by only $0.3\,°\mathrm{C}$.

Temperature of liquid nitrogen When the bulb is cooled down to the temperature of liquid nitrogen the sensor read $U_N = (-5.070 \pm 0.005)\,\mathrm{mV}$, which means that it is at a pressure of $(19\,577 \pm 5)\,\mathrm{Pa}$. Using (9) we get an approximate value of

$$t_{N0} = (-196.6 \pm 0.1)\,°\mathrm{C},$$

which is off by about $0.8\,°\mathrm{C}$ from the literature value of $195.8\,°\mathrm{C}$. If we again consider the parasitic volume and thermal expansion and use (10) and (11) we get

$$t_N = (-196.8 \pm 0.1)\,°\mathrm{C},$$

which is a bit further off by $1\,°\mathrm{C}$. Given that t_{N0} and t_N are only $0.2\,°\mathrm{C}$ apart, each with an error of $0.1\,°\mathrm{C}$, we can conclude that the parasitic volume and the thermal expansion don't contribute a lot to the error.

4 Conclusion

We accurately calibrated the pressure sensor by measuring two known pressures and then linearly interpolating between the two. This allowed us to measure the pressure of helium at two know temperatures, which with the ideal gas law, allowed us to determine an absolute zero temperature. If we further considered the parasitic volume of the throat of the bulb and its thermal expansion we could further improve our result of $(-272.9 \pm 0.1)\,°\mathrm{C}$, which is only $0.3\,°\mathrm{C}$ off the the real value of $-273.15\,°\mathrm{C}$. Finally we used the setup as a thermometer and measured the temperature of boiling liquid nitrogen. This time, considering the parasitic volume and the thermal expansion did not improve the result, but slightly worsened it by an amount comparable to the error of the result. Our measured and corrected value was $(-196.8 \pm 0.1)\,°\mathrm{C}$, which is $1\,°\mathrm{C}$ off the real value of $-195.8\,°\mathrm{C}$.