63 - Alternating Current and *RLC*

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Abstract

In this lab we investigate two AC networks. Firstly, for an *RC* low pass filter, we take measurements of the output voltage U_{out} and the phase shift φ between the input and output signals. From these values we calculate the limiting frequency $\omega_{\text{gr}}^{\text{exp}} = 10\,400\,\text{s}^{-1}$ and the roll-off $-10\,\text{dB/dec}$. The second network is an *RLC* parallel resonant circuit, for which we again measure U_{out} and φ and then calculate the resonant frequency $\omega_0^{\text{exp}} = 33\,100\,\text{s}^{-1}$ and the quality factor $Q = 2.64$.

Contents

1 Introduction

RLC-circuits (or networks) are built using resistors, with resistance R, inductors, with inductance L, and capacitors with capacity C. The voltage drop across these three components is given by the following equations:

$$U(t) = RI(t) \qquad U(t) = L\frac{\mathrm{d}I(t)}{\mathrm{d}t} \qquad \frac{\mathrm{d}U(t)}{\mathrm{d}t} = \frac{1}{C}I(t).$$

When working with alternating currents it is best to use complex notation, which allows us to encode the phase shift between the voltage and the current in the complex variables. Further we'll introduce the impedance Z, which allows us to formulate a law analogous to Ohm's law: $U(t) = ZI(t)$. The impedances of the resistor, the inductor and the capacitor are as follows:

$$Z_R = R \qquad Z_L = i\omega L \qquad Z_C = \frac{1}{i\omega C}.$$

Across an inductor, the voltage will be shifted forwards by $\pi/2$ in relation to the current, and across a capacitor the voltage will be $\pi/2$ behind. A resistor induces no phase shift.

Firstly we investigated an RC low pass filter as shown in figure 1. According to theory the ratio between the output and input voltages is given by

$$\frac{U_{\text{out}}}{U_{\text{in}}} = \frac{1}{\sqrt{\omega^2 C^2 R^2 + 1}}. \tag{1}$$

The *limiting frequency* ω_{gr} of such a filter is defined as the frequency at which $\frac{U_{\text{out}}}{U_{\text{in}}} = \frac{1}{\sqrt{2}}$. For our circuit this is given by:

$$\omega_{\text{gr}} = \frac{1}{RC}. \tag{2}$$

The slope of the curve at ω_{gr} in figure 3, which plots $\frac{U_{\text{out}}}{U_{\text{in}}}$ against ω on a logarithmic scale, is called the *roll-off* and is equivalent to the sharpness of the filter. The roll-off is usually given in decibels per decade (dB/dec) or decibels per octave (dB/oct). Further, a phase shift φ occurs between U_{in} and U_{out}, which behaves as follows:

$$\varphi = -\tan^{-1}\left(\frac{\omega}{\omega_{\text{gr}}}\right). \tag{3}$$

For $\omega \to 0$, φ tends to $0°$, for $\omega \to \infty$, φ approaches $90°$ and at ω_{gr} the phase shift should exactly be $45°$. Finally we looked at the phase shift between the voltage and the current across the capacitor. According to theory this should be independent of the frequency.

The second network we investigated is an RLC parallel resonant circuit shown in figure 2. According to theory the ratio between the output and input voltages of this circuit is given by

$$\frac{U_{\text{out}}}{U_{\text{in}}} = \left|\frac{Z_2}{R + Z_2}\right|, \quad \text{where } Z_2 = \frac{1}{i\omega C + \frac{1}{i\omega L}}. \tag{4}$$

The *resonant frequency* is given by

$$\omega_0 = \frac{1}{\sqrt{LC}}. \tag{5}$$

In the graph in figure 5, which again plots $\frac{U_{\text{out}}}{U_{\text{in}}}$ against ω on a logarithmic scale, ω_0 is at the peak of the curve, where $\frac{U_{\text{out}}}{U_{\text{in}}}$ is maximal. The range of frequencies for which $\frac{U_{\text{out}}}{U_{\text{in}}} \geq \frac{1}{\sqrt{2}}$ is called the *bandwidth* $\Delta\omega$. In a parallel resonant circuit this is given by

$$\Delta\omega = \frac{R}{L}. \tag{6}$$

Depending on the application, it is desirable to have a narrow or a large bandwidth. A narrow bandwidth corresponds to a sharply defined resonance. A measure of this is the *quality factor* (Q-factor), which is defined as

$$Q = \frac{\omega_0}{\Delta\omega} = R\sqrt{\frac{C}{L}}. \tag{7}$$

Lastly, we again have a phase shift given by

$$\varphi = \tan^{-1}\left[R\left(\omega C - \frac{1}{\omega L}\right)\right]. \tag{8}$$

2 Experiment

2.1 RC Low Pass Filter

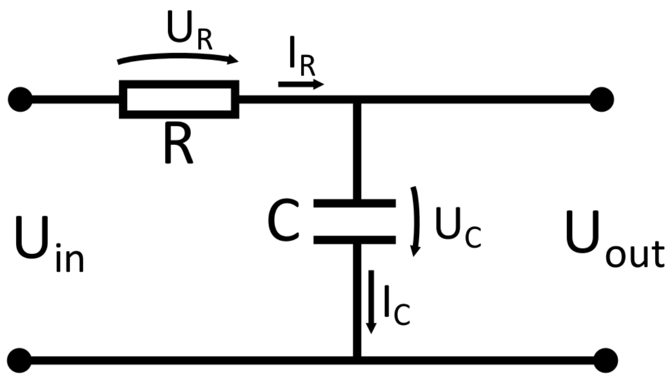

Figure 1: The *RC* low pass filter investigated in the first part of the experiments. The values used were: $R = (1.00 \pm 0.01)\,\text{k}\Omega$, $C = (100.0 \pm 0.5)\,\text{nF}$ and $U_{\text{in}} = (1.00 \pm 0.05)\,\text{V}$.

We built the circuit shown in figure 1 on a bread board and using off the shelf components. The values used were $R = (1.00 \pm 0.01)\,\text{k}\Omega$, $C = (100.0 \pm 0.5)\,\text{nF}$. A signal generator was then hooked up to U_{in} generating a sine wave at a peak-to-peak voltage of $(1.00 \pm 0.05)\,\text{V}$, which will be held fixed throughout all experiments. To perform our measurements we set up an oscilloscope to measure U_{in} and U_{out}.

We varied the input frequency f from 10 Hz up to 100 kHz and took 11 measurements of the peak-to-peak output voltage U_{out} and of the phase shift φ between U_{in} and U_{out}. We then fitted these measurements with (1) and (3) to determine and experimental value for the limiting frequency ω_{gr} and the phase shift φ at ω_{gr}. Finally we measured the phase shift between the current I_C and the voltage U_C across the capacitor for some frequencies around ω_{gr}. To do this we used the fact that the current through R and C are the: $I_R = I_C$. Thus, we only had to measure the voltage drop across the resistor which is equal to $U_{\text{in}} - U_{\text{out}} = U_R$. Therefore we have $I_C \propto U_{\text{in}} - U_{\text{out}}$ and $U_C = U_{\text{out}}$, which means we don't have to change our probe set up.

2.2 RLC Parallel Resonant Circuit

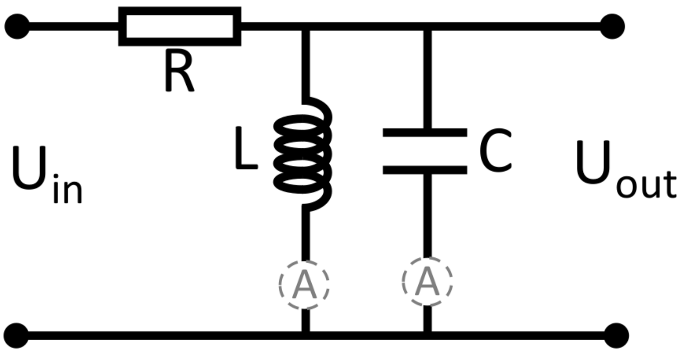

Figure 2: The RLC parallel resonant circuit investigated in the second part of the experiments. The values used were: $R = (1.00 \pm 0.01)\,\text{k}\Omega$, $L = (10.00 \pm 0.05)\,\text{mH}$, $C = (100.0 \pm 0.5)\,\text{nF}$ and $U_{\text{in}} = (1.00 \pm 0.05)\,\text{V}$.

To build the RLC parallel resonant circuit shown in figure 2, we simply had to add an inductor $L = (10.00 \pm 0.05)\,\text{mH}$ to the circuit from the previous experiment.

Again, we varied the input frequency f between 1 kHz and 24 kHz and took 19 measurements of the peak-to-peak output voltage U_{out} and the phase shift φ between U_{in} and U_{out}. Fitting these measurements with (4), we then determined an experimental value for the resonance frequency ω_0, the bandwidth $\Delta\omega$ and the quality-factor Q. Finally we measured the resonant frequency ω_0 for 3 different capacitances $C = 10, 100, 1000\,\text{nF}$ and 2 different inductances $L = 1, 10\,\text{mH}$, giving a total of 6 resonant frequencies.

3 Results

3.1 RC Low Pass Filter

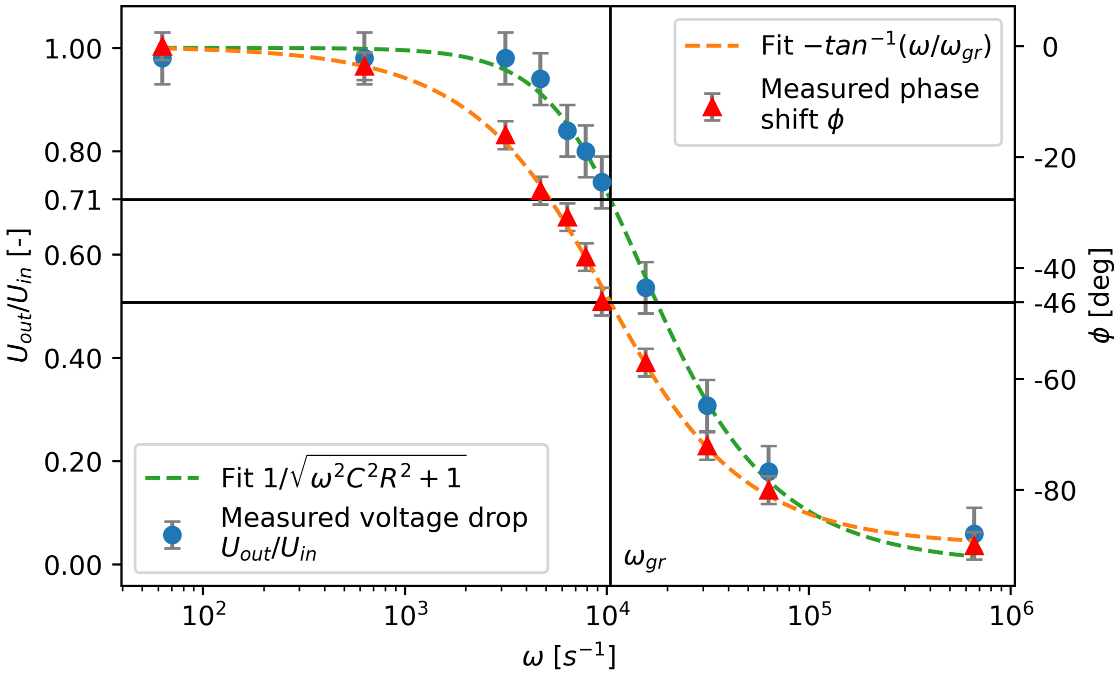

Figure 3: Semi-logarithmic plot of all measurements of the RC low pass filter. In blue are the measurements of U_{out} plotted as a fraction $\frac{U_{\text{out}}}{U_{\text{in}}}$ of the constant input voltage (1.00 ± 0.05) V. The errors ($\Delta U_{\text{out}} = 0.05$ V, $\Delta\varphi = 2.5^\circ$)are estimated from the fluctuations we saw, when taking the measurements. These measurements have been fitted with equation (1). In red are the measurements of the phase shift, also fitted with equation (3).

The graph in figure 3 shows all measurements of the RC low pass filter. The measured output voltages U_{out} are plotted as a ratio $\frac{U_{\text{out}}}{U_{\text{in}}}$ of the constant input voltage $U_{\text{in}} = (1.00 \pm 0.05)$ V. Fitting the voltage measurements with equation (1) gives us the capacitance and resistance values shown in table 1:

	R [kΩ]	C [nF]
Real	1 ± 0.01	100 ± 0.5
Fit	0.942	102

Table 1: Fit parameters of the fit for the measured voltage drop in figure 3, according to equation (1). The real values are the labels of the components used.

This allows us to experimentally determine the limiting frequency $\omega_{\text{gr}}^{\text{exp}}$ and compare it to the theoretical prediction $\omega_{\text{gr}}^{\text{theo}}$ using formula (2):

$$\omega_{\text{gr}}^{\text{exp}} = 10\,400\,\text{s}^{-1} \qquad \text{and} \qquad \omega_{\text{gr}}^{\text{theo}} = (10\,000 \pm 100)\,\text{s}^{-1}.$$

From the graph in figure 3 we can see that for $\omega \ll \omega_{\text{gr}}$, the fraction $\frac{U_{\text{out}}}{U_{\text{in}}}$ goes to one, which means that the circuit behaves as if the positive and the negative poles of the input and output respectively were simply connected together. This makes sense, as the capacitor at low frequencies effectively blocks any current from flowing through it. For $\omega \gg \omega_{\text{gr}}$ on the other hand, the capacitor has virtually no resistance and creates a short circuit on the output side. Therefore $\frac{U_{\text{out}}}{U_{\text{in}}}$ goes to zero.

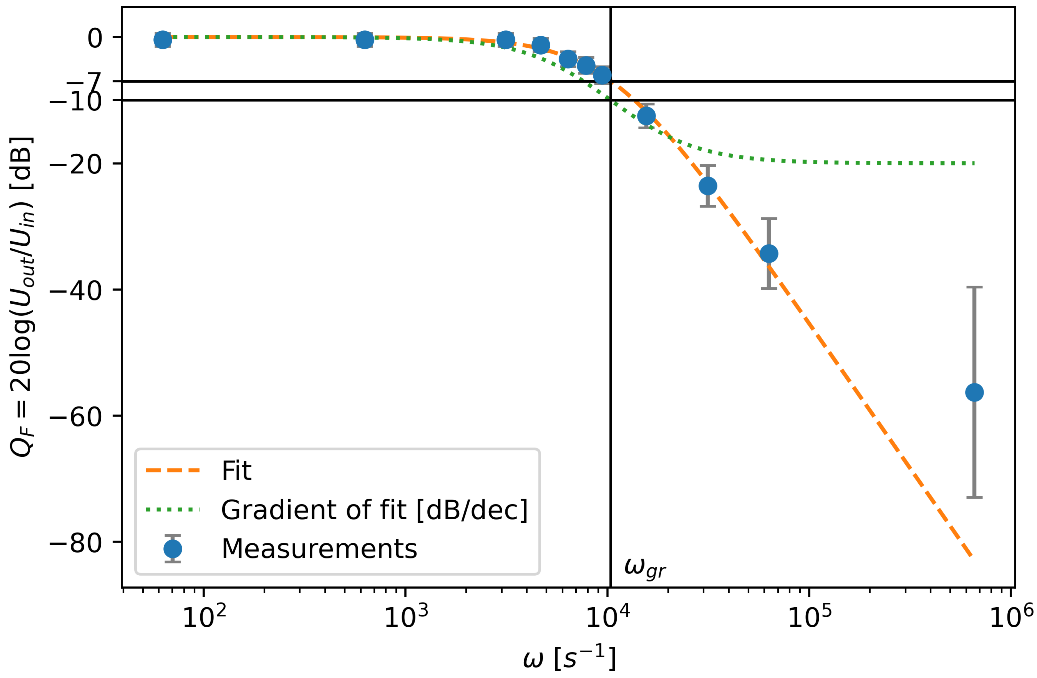

Figure 4: The same measurements on the voltage drop from figure 3, plotted in decibels, given by $Q_F = 20\log\left(\frac{U_{\text{out}}}{U_{\text{in}}}\right)$. The fit line uses the fit parameters from table 1, the same as in figure 3. The green dotted line shows the gradient of the fit and is used to determine the roll-off.

To calculate the roll-off in appropriate units, we can look at figure 4, which plots the same data from figure 3 in decibels. We can again see a fit with the same fit parameters from table 1. The gradient of the fit at the limiting frequency ω_{gr} gives us the roll-off in decibels per decade:

$$\text{Roll-off:} \quad -10\,\text{dB/dec.}$$

The measured phase shift φ can also be seen in figure 3 along with a fit according to equation (3), which gives a fit value for the limiting frequency of

$$\omega_{\mathrm{gr}}^{\mathrm{fit}} = 9990\,\mathrm{s}^{-1}.$$

Lastly, the phase shift between the current I_C and the voltage U_C over the capacitor was measured to be 90° for all frequencies in a range from 6 kHz up to 60 kHz, including ω_{gr}.

3.2 RLC Parallel Resonant Circuit

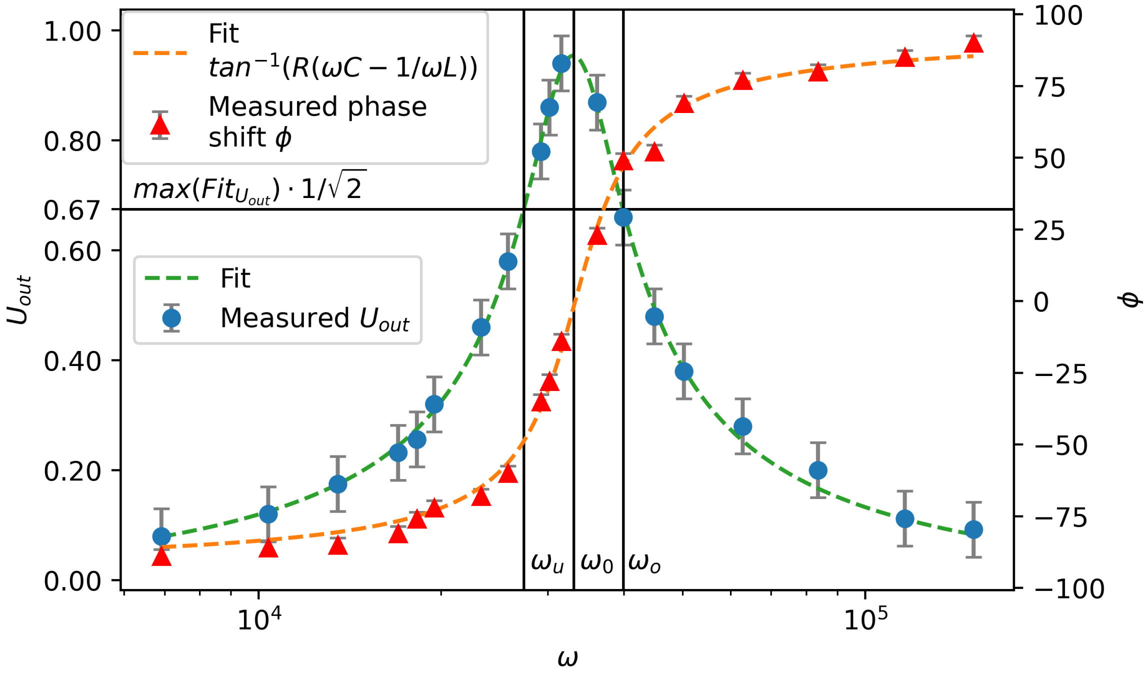

Figure 5: Semi-logarithmic plot of the measurements on the RLC circuit shown in figure 2. The output voltage U_{out} was fitted with (4) and the phase shift φ was fitted with (8). The errors ($\Delta U_{\mathrm{out}} = 0.05\,\mathrm{V}$, $\Delta\varphi = 2.5°$) are estimated from the fluctuations we saw, when taking the measurements. The resonant frequency ω_0, and the lower and upper bound ω_u and ω_o of the bandwidth are also shwon in the plot.

Figure 5 plots the voltage U_{out} and phase shift φ measurements of the RLC resonant circuit from figure 2 for various frequencies ω on a logarithmic scale. Both are measurements are fit with equations (4) and (8) respectively. Fit parameters are shown in table 2:

From these fits we can determine the resonant frequency and compare it to the theoretical prediction according to (5):

$$\omega_0^{\mathrm{exp}} = 33\,100\,\mathrm{s}^{-1} \qquad \text{and} \qquad \omega_0^{\mathrm{theo}} = (31\,600 \pm 100)\,\mathrm{s}^{-1}.$$

	U_{in} [V]	R [kΩ]	L [mH]	C [nF]
Real	1 ± 0.05	1 ± 0.01	10 ± 0.05	100 ± 0.5
Voltage fit	0.954 ± 0.01	–	11.5 ± 0.3	80 ± 1
Phase fit	–	0.860	8.9	102

Table 2: Fit parameters for the voltage fit according to equation (4) and the phase fit according to equation (8), both seen in figure 5. The real values are the labels of the components used.

Further we can determine the bandwidth from the graph to be $\Delta\omega = 12\,500\,\text{s}^{-1}$, which allows us to calculate an experimental value for the Q-factor and again compare it to theory 7:

$$Q^{\text{exp}} = 2.64 \qquad \text{and} \qquad Q^{\text{theo}} = 3.16 \pm 0.03.$$

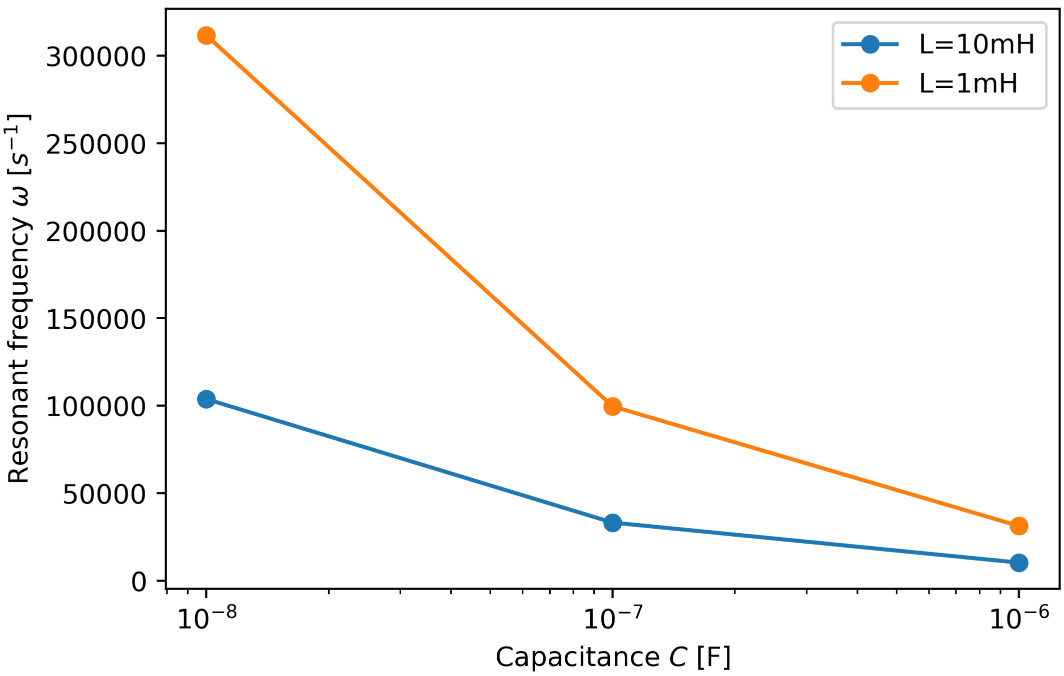

Figure 6: The measured resonant frequencies of the RLC circuit shown in figure 2 for various inductances ($L = 1, 10\,\text{mH}$) and capacitances ($C = 10, 100, 1000\,\text{nF}$).

Next, we measured the resonant frequency of the RLC circuit for various inductances ($L = 1, 10\,\text{mH}$) and capacitances ($C = 10, 100, 1000\,\text{nF}$), the results of which are plotted in figure 6.

Lastly, we fed the RLC circuit with triangle and square waves. When observing the output signal on the oscilloscope, we could see that the shape of the wave has greatly changed. The reason for this, is that the non-sinusoidal signals are composed of many different frequencies. Now if that signal passes through an *RLC* network some frequencies get filtered out more than others, which changes the shape of the curve. This change also depends on the base frequency of the input signal, which is what we observed in our experiments.

4 Discussion

For both networks that were investigated we got very reasonable results. Starting with the *RC* low pass filter, all fit parameters were within 5% of the real or theoretical values. The only anomaly is the roll-off, which we determined to be $-10\,\mathrm{dB/dec}$, which is half of the expected $-20\,\mathrm{dB/dec}$ of a first order filter.

As for the *RLC* parallel resonant circuit, the fit parameters deviated by about 10-20% from the real values. This led an experimental resonant frequency ω_0, which is still within 5% of the predicted theoretical value. Though, The quality factor Q deviated by about 17% from the theoretical value.

Finally, we saw in our last measurement, that the resonant frequency of an *RLC* circuit decreases, as the Capacitance and the inductance increase, which is also in accordance to what we expect.

5 Conclusion

In conclusion we measured the limiting frequency of the RC filter in figure 1 to be $\omega_{\mathrm{gr}}^{\mathrm{exp}} = 10\,400\,\mathrm{s}^{-1}$, which is within 5% of the theoretical value. The roll-off was measured to be $-10\,\mathrm{dB/dec}$, half of what we would expect from a first-order filter. For the *RLC* parallel resonant circuit from figure 2 we measured a limiting frequency of $\omega_0^{\mathrm{exp}} = 33\,100\,\mathrm{s}^{-1}$, which is withing 5% of the theoretical value. The Q-factor was measured to be 2.64, compared to the theoretical prediction of 3.16. Finally we showed that the resonant frequency of an *RLC* circuit increases, as the capacitance and the inductance increase.

References

[1] https://ap.phys.ethz.ch/Anleitungen/Bilingual/63_Manual.pdf

ETH Z	**Physikalisches Praktikum**	Datum 28.3.22
Name 1 Manuel Antoinette	Name 2	
63	**Wechselströme und R, L, C**	Platz-Nr.

Hausaufgaben

1. Scheitelwerte und Effektivwerte

☒ Ich kenne den Unterschied zwischen Scheitelwert und Effektivwert. Die Umrechnung zwischen den beiden Werten lautet: $1/\sqrt{2}$

2. Filter

Filter aus Fig. 6 f) : U_{out} / U_{in} = $\frac{i\omega RC}{i\omega RC+1}$

Es handelt sich somit um einen Hoch-passfilter

Filter aus Fig. 6 a) : U_{out} / U_{in} = $\frac{1}{i\omega RC+1}$

Es handelt sich somit um einen Tief-passfilter

3. (freiwillig) **Unterschied RC- und LC-Filter**

Argumentieren Sie, welcher der beiden Filter eine steilere Flanke hat!

Teil I – Frequenzabhängige Spannungsteiler und Filter

1. Tiefpassfilter

C = 100 ± 0.5 nF R = 1 ± 0.01 kΩ U_{in} = 1. ± 0.05 V

[Hz]	f	9.96, 100, 500, 745, 1014, 1250, 1500, 2480, 5000, 10060, 105000
	ω	$2\pi f$
[mV]	U_{out}	980, 980, 980, 940, 840, 800, 740, 536, 308, 180, 60
[deg]	φ	0, -3.6, -16, -26, -30.8, -38, -46, -57, -72, -80, -90

Grafische Darstellung von U_{out} / U_{in} als Funktion von $\log(\omega)$.

2. Grenzfrequenz

Aus Plot: ω_{gr}^{exp} = 10400 s^{-1} Theoretische Vorhersage: ω_{gr}^{theo} = 10000 ± 100 s^{-1}

3. Grenzfälle

Diskutieren Sie den Verlauf der Kurve, v.a. für die Grenzfälle $\omega \ll \omega_{gr}$ und $\omega \gg \omega_{gr}$. Wie erklären Sie sich das Aussehen der Kurve für die Grenzfälle?

see report

4. Roll-Off

Der Roll-Off beträgt ca. -10 dB/dec *[Einheiten angeben!]*

5. Phasenverschiebung zwischen U_{in} und U_{out}

Zeichnen Sie die Phasenverschiebung φ zwischen U_{in} und U_{out} als Funktion von $\log(\omega)$, idealerweise in den gleichen Plot wie Aufgabe 1.

Die Phasenverschiebung bei der Grenzfrequenz beträgt $\varphi(\omega_{gr})$ = -46°

6. Phasenverschiebung zwischen Strom und Spannung über dem Kondensator

Phasenverschiebungen zwischen Strom und Spannung über dem Kondensator:

φ bei ω_{gr}: 90°

Bei weiteren Frequenzen:

90° for all tested frequencies

Teil II – RLC-Parallelkreis

1. Paralleler RLC-Schwingkreis

C = 100 ± 0.5 nF R = 1 ± 0.01 kΩ L = 10 ± 0.05 mH U_{in} = 1. ± 0.05 V

Aus diesen Werten folgt: ω_0^{theo} = 31600 ± 1005⁻¹

[kHz]	f	5.02, 5.75, 6.35, 7.15, 8, 10, 13.3, 18.5, 24, 4.65, 4.8, 4.1, 3.7, 3.1, 2.9, 2.7, 2.15, 1.65, 1.1
	ω	2πf
[mV]	U_{out}	940, 869, 660, 480, 380, 280, 200, 112, 92, 780, 860, 580, 460, 320, 256, 232, 175, 120, 80
[deg]	φ	-14, 23, 49, 52, 69, 77, 80, 85, 90, -35, -28, -60, -68, -72, -76, -81, -85, -86, -89

2. Grafische Darstellung von U_{out} als Funktion von ω.

3. Güte Q des Resonanzkreises (per Fit und/oder grafisch)

Q_{graf} = 2.64 ± ....

Q_{fit} = 3.16 ± 0.03 (nur D-PHYS)

4. Strom durch den Schwingkreis

C = ... R = ... L = ... U_{in} = ...

f	
ω	
I_R	
φ	

Grafische Darstellung von φ als Funktion von ω.

5. (nur D-PHYS) **Resonanzfrequenz für verschiedene L und C**

	L =	C =	f_0 =
i.	10	10	16.52
ii.	10	100	5.285
iii.	10	1000	1.645
iv.	1	1000	4.975
v.	1	100	15.86
vi.	1	10	49.6
	[mH]	[nF]	[kHz]

Grafische Darstellung von ω_0 als sinnvolle Funktion $f(L, C)$.

6. (nur D-PHYS) **Rechtecks- und Dreieckssignal**

Speisen Sie den RLC-Kreis mit einem Rechtecks- bzw. Dreieckssignal, und verändern Sie U_{in}. Was beobachten Sie?

Stichworte: Ringdown, Fouriertransformation des Eingangssignals

7. (freiwillig) **Ströme im Parallelkreis**

Messen Sie die Ströme I_C und I_L mit dem Multimeter. Was beobachten Sie? Wie gross werden die Ströme relativ zum Strom über dem Widerstand I_R?